Physics 4B

Solutions to Chapter 27 HW

Chapter 27: Questions: 2, 8, 10
Problems: 1, 23, 33, 45, 48, 61, 63, 72, 92

Question 27-2
(a) no (b) yes (c) all tie

Question 27-8
60 μC

Question 27-10
1, *c*; 2, *a*; 3, *d*; 4, *b*

Problem 27-1
(a) Let *i* be the current in the circuit and take it to be positive if it is to the left in R_1. We use Kirchhoff's loop rule: $\varepsilon_1 - iR_2 - iR_1 - \varepsilon_2 = 0$. We solve for *i*:

$$i = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{12\ \text{V} - 6.0\ \text{V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\ \text{A}.$$

A positive value is obtained, so the current is counterclockwise around the circuit.

If *i* is the current in a resistor *R*, then the power dissipated by that resistor is given by $P = i^2R$.

(b) For R_1, $P_1 = i^2R_1 = (0.50\ \text{A})^2(4.0\ \Omega) = 1.0$ W,

(c) and for R_2, $P_2 = i^2R_2 = (0.50\ \text{A})^2\ (8.0\ \Omega) = 2.0$ W.

If *i* is the current in a battery with emf ε, then the battery supplies energy at the rate $P = i\varepsilon$ provided the current and emf are in the same direction. The battery absorbs energy at the rate $P = i\varepsilon$ if the current and emf are in opposite directions.

(d) For ε_1, $P_1 = i\varepsilon_1 = (0.50\ \text{A})(12\ \text{V}) = 6.0$ W

(e) and for ε_2, $P_2 = i\varepsilon_2 = (0.50\ \text{A})(6.0\ \text{V}) = 3.0$ W.

(f) In battery 1 the current is in the same direction as the emf. Therefore, this battery supplies energy to the circuit; the battery is discharging.

(g) The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.

Problem 27-23

Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is upward.

(a) When the loop rule is applied to the lower loop, the result is

$$\varepsilon_2 - i_1 R_1 = 0 .$$

The equation yields

$$i_1 = \frac{\varepsilon_2}{R_1} = \frac{5.0 \text{ V}}{100\,\Omega} = 0.050 \text{ A}.$$

(b) When it is applied to the upper loop, the result is

$$\varepsilon_1 - \varepsilon_2 - \varepsilon_3 - i_2 R_2 = 0 .$$

The equation gives

$$i_2 = \frac{\varepsilon_1 - \varepsilon_2 - \varepsilon_3}{R_2} = \frac{6.0 \text{ V} - 5.0 \text{ V} - 4.0 \text{ V}}{50\,\Omega} = -0.060 \text{ A} ,$$

or $|i_2| = 0.060$ A. The negative sign indicates that the current in R_2 is actually downward.

(c) If V_b is the potential at point *b*, then the potential at point *a* is $V_a = V_b + \varepsilon_3 + \varepsilon_2$, so

$$V_a - V_b = \varepsilon_3 + \varepsilon_2 = 4.0 \text{ V} + 5.0 \text{ V} = 9.0 \text{ V}.$$

Problem 27-33

First, we note in V_4, that the voltage across R_4 is equal to the sum of the voltages across R_5 and R_6:

$$V_4 = i_6(R_5 + R_6) = (1.40 \text{ A})(8.00\ \Omega + 4.00\ \Omega) = 16.8 \text{ V}.$$

The current through R_4 is then equal to $i_4 = V_4/R_4 = 16.8 \text{ V}/(16.0\ \Omega) = 1.05$ A.

By the junction rule, the current in R_2 is

$$i_2 = i_4 + i_6 = 1.05 \text{ A} + 1.40 \text{ A} = 2.45 \text{ A},$$

so its voltage is $V_2 = (2.00\ \Omega)(2.45 \text{ A}) = 4.90$ V.

The loop rule tells us the voltage across R_3 is $V_3 = V_2 + V_4 = 21.7$ V (implying that the current through it is $i_3 = V_3/(2.00\ \Omega) = 10.85$ A).

The junction rule now gives the current in R_1 as $i_1 = i_2 + i_3 = 2.45$ A + 10.85 A = 13.3 A, implying that the voltage across it is $V_1 = (13.3\text{ A})(2.00\ \Omega) = 26.6$ V. Therefore, by the loop rule,

$$\varepsilon = V_1 + V_3 = 26.6\text{ V} + 21.7\text{ V} = 48.3\text{ V}.$$

Problem 27-45

(a) We note that the R_1 resistors occur in series pairs, contributing net resistance $2R_1$ in each branch where they appear. Since $\varepsilon_2 = \varepsilon_3$ and $R_2 = 2R_1$, from symmetry we know that the currents through ε_2 and ε_3 are the same: $i_2 = i_3 = i$. Therefore, the current through ε_1 is $i_1 = 2i$. Then from $V_b - V_a = \varepsilon_2 - iR_2 = \varepsilon_1 + (2R_1)(2i)$ we get

$$i = \frac{\varepsilon_2 - \varepsilon_1}{4R_1 + R_2} = \frac{4.0\text{ V} - 2.0\text{ V}}{4(1.0\ \Omega) + 2.0\ \Omega} = 0.33\text{ A}.$$

Therefore, the current through ε_1 is $i_1 = 2i = 0.67$ A.

(b) The direction of i_1 is downward.

(c) The current through ε_2 is $i_2 = 0.33$ A.

(d) The direction of i_2 is upward.

(e) From part (a), we have $i_3 = i_2 = 0.33$ A.

(f) The direction of i_3 is also upward.

(g) $V_a - V_b = -iR_2 + \varepsilon_2 = -(0.333\text{ A})(2.0\ \Omega) + 4.0\text{ V} = 3.3$ V.

Problem 27-48

(a) We use $P = \varepsilon^2/R_{eq}$, where

$$R_{eq} = 7.00\ \Omega + \frac{(12.0\ \Omega)(4.00\ \Omega)R}{(12.0\ \Omega)(4.0\ \Omega) + (12.0\ \Omega)R + (4.00\ \Omega)R}.$$

Put $P = 60.0$ W and $\varepsilon = 24.0$ V and solve for R: $R = 19.5\ \Omega$.

(b) Since $P \propto R_{eq}$, we must minimize R_{eq}, which means $R = 0$.

(c) Now we must maximize R_{eq}, or set $R = \infty$.

(d) Since $R_{eq,\,min} = 7.00\ \Omega$, $P_{max} = \varepsilon^2/R_{eq,\,min} = (24.0\text{ V})^2/7.00\ \Omega = 82.3$ W.

(e) Since $R_{eq,\,max} = 7.00\ \Omega + (12.0\ \Omega)(4.00\ \Omega)/(12.0\ \Omega + 4.00\ \Omega) = 10.0\ \Omega$,

$$P_{min} = \varepsilon^2/R_{eq,\,max} = (24.0\text{ V})^2/10.0\ \Omega = 57.6\text{ W}.$$

Problem 27-61

(a) The voltage difference V across the capacitor is $V(t) = \varepsilon(1 - e^{-t/RC})$. At $t = 1.30\ \mu$s we have $V(t) = 5.00$ V, so 5.00 V = (12.0 V)$(1 - e^{-1.30\ \mu s/RC})$, which gives

$$\tau = (1.30\ \mu\text{ s})/\ln(12/7) = 2.41\ \mu\text{s}.$$

(b) The capacitance is $C = \tau/R = (2.41\ \mu\text{s})/(15.0\ \text{k}\Omega) = 161$ pF.

Problem 27-63

At $t = 0$ the capacitor is completely uncharged and the current in the capacitor branch is as it would be if the capacitor were replaced by a wire. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces $i_1 = i_2 + i_3$, the loop rule applied to the left-hand loop produces

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0 ,$$

and the loop rule applied to the right-hand loop produces

$$i_2 R_2 - i_3 R_3 = 0 .$$

Since the resistances are all the same we can simplify the mathematics by replacing R_1, R_2, and R_3 with R.

(a) Solving the three simultaneous equations, we find

$$i_1 = \frac{2\varepsilon}{3R} = \frac{2\left(1.2\times10^3\ \text{V}\right)}{3\left(0.73\times10^6\ \Omega\right)} = 1.1\times10^{-3}\ \text{A} ,$$

(b) $i_2 = \dfrac{\varepsilon}{3R} = \dfrac{1.2\times10^3\ \text{V}}{3\left(0.73\times10^6\ \Omega\right)} = 5.5\times10^{-4}\ \text{A}$, and

(c) $i_3 = i_2 = 5.5\times10^{-4}\ \text{A}$.

At $t = \infty$ the capacitor is fully charged and the current in the capacitor branch is 0. Thus, $i_1 = i_2$, and the loop rule yields

$$\varepsilon - i_1 R_1 - i_1 R_2 = 0 .$$

(d) The solution is

$$i_1 = \frac{\varepsilon}{2R} = \frac{1.2\times10^3\ \text{V}}{2\left(0.73\times10^6\ \Omega\right)} = 8.2\times10^{-4}\ \text{A}.$$

(e) $i_2 = i_1 = 8.2\times10^{-4}\ \text{A}$.

(f) As stated before, the current in the capacitor branch is $i_3 = 0$.

We take the upper plate of the capacitor to be positive. This is consistent with current flowing into that plate. The junction equation is $i_1 = i_2 + i_3$, and the loop equations are

$$\varepsilon - i_1 R - i_2 R = 0$$

$$-\frac{q}{C} - i_3 R + i_2 R = 0 \ .$$

We use the first equation to substitute for i_1 in the second and obtain $\varepsilon - 2i_2 R - i_3 R = 0$. Thus $i_2 = (\varepsilon - i_3 R)/2R$. We substitute this expression into the third equation above to obtain

$$-(q/C) - (i_3 R) + (\varepsilon/2) - (i_3 R/2) = 0.$$

Now we replace i_3 with dq/dt to obtain

$$\frac{3R}{2}\frac{dq}{dt} + \frac{q}{C} = \frac{\varepsilon}{2} \ .$$

This is just like the equation for an RC series circuit, except that the time constant is $\tau = 3RC/2$ and the impressed potential difference is $\varepsilon/2$. The solution is

$$q = \frac{C\varepsilon}{2}\left(1 - e^{-2t/3RC}\right) \ .$$

The current in the capacitor branch is

$$i_3(t) = \frac{dq}{dt} = \frac{\varepsilon}{3R} e^{-2t/3RC} \ .$$

The current in the center branch is

$$i_2(t) = \frac{\varepsilon}{2R} - \frac{i_3}{2} = \frac{\varepsilon}{2R} - \frac{\varepsilon}{6R} e^{-2t/3RC} = \frac{\varepsilon}{6R}\left(3 - e^{-2t/3RC}\right)$$

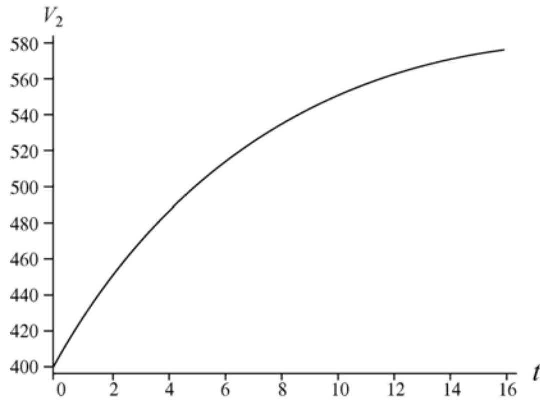
and the potential difference across R_2 is

$$V_2(t) = i_2 R = \frac{\varepsilon}{6}\left(3 - e^{-2t/3RC}\right) .$$

(g) For $t = 0$, $e^{-2t/3RC} = 1$ and $V_2 = \varepsilon/3 = \left(1.2\times10^3\ \text{V}\right)/3 = 4.0\times10^2\ \text{V}$.

(h) For $t = \infty$, $e^{-2t/3RC} \to 0$ and $V_2 = \varepsilon/2 = \left(1.2\times20^3\ \text{V}\right)/2 = 6.0\times10^2\ \text{V}$.

(i) A plot of V_2 as a function of time is shown in the following graph.



Problem 27-72

(a) The four resistors R_1, R_2, R_3, and R_4 on the left reduce to

$$R_{eq} = R_{12} + R_{34} = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = 7.0\ \Omega + 3.0\ \Omega = 10\ \Omega.$$

With $\varepsilon = 30$ V across R_{eq} the current there is $i_2 = 3.0$ A.

(b) The three resistors on the right reduce to

$$R'_{eq} = R_{56} + R_7 = \frac{R_5 R_6}{R_5 + R_6} + R_7 = \frac{(6.0\ \Omega)(2.0\ \Omega)}{6.0\ \Omega + 2.0\ \Omega} + 1.5\ \Omega = 3.0\ \Omega.$$

With $\varepsilon = 30$ V across R'_{eq} the current there is $i_4 = 10$ A.

(c) By the junction rule, $i_1 = i_2 + i_4 = 13$ A.

(d) By symmetry, $i_3 = \frac{1}{2} i_2 = 1.5$ A.

(e) By the loop rule (proceeding clockwise),

$$30\text{V} - i_4(1.5\ \Omega) - i_5(2.0\ \Omega) = 0$$

readily yields $i_5 = 7.5$ A.

Problem 27-92

The equivalent resistance of the series pair of $R_3 = R_4 = 2.0\ \Omega$ is $R_{34} = 4.0\ \Omega$, and the equivalent resistance of the parallel pair of $R_1 = R_2 = 4.0\ \Omega$ is $R_{12} = 2.0\ \Omega$. Since the voltage across R_{34} must equal that across R_{12}:

$$V_{34} = V_{12} \quad \Rightarrow \quad i_{34} R_{34} = i_{12} R_{12} \quad \Rightarrow \quad i_{34} = \frac{1}{2} i_{12}$$

This relation, plus the junction rule condition $I = i_{12} + i_{34} = 6.00$ A, leads to the solution $i_{12} = 4.0$ A . It is clear by symmetry that $i_1 = i_{12} / 2 = 2.00$ A .