Physics 4B

Solutions to Chapter 28 HW

Chapter 28: Questions: 4, 6, 10
Problems: 4, 11, 17, 33, 36, 47, 49, 51, 60, 74

Question 28-4
into page: *a*, *d*, *e*; out of page: *b*, *c*, *f* (the particle is negatively charged)

Question 28-6
2, 5, 6, 9, 10

Question 28-10
1*i*, 2*e*, 3*c*, 4*a*, 5*g*, 6*j*, 7*d*, 8*b*, 9*h*, 10*f*, 11*k*

Problem 28-4
(a) We use Eq. 28-3:

$$F_B = |q|\, vB \sin\phi = (+\,3.2\times10^{-19}\text{ C})\,(550\text{ m/s})\,(0.045\text{ T})\,(\sin 52°) = 6.2\times10^{-18}\text{ N}.$$

(b) The acceleration is

$$a = F_B/m = (6.2\times10^{-18}\text{ N})\,/\,(6.6\times10^{-27}\text{ kg}) = 9.5\times10^{8}\text{ m/s}^2.$$

(c) Since it is perpendicular to $\vec{v}, \vec{F}_B$ does not do any work on the particle. Thus from the work-energy theorem both the kinetic energy and the speed of the particle remain unchanged.

Problem 28-11
Since the total force given by $\vec{F} = e\left(\vec{E} + \vec{v}\times\vec{B}\right)$ vanishes, the electric field $\vec{E}$ must be perpendicular to both the particle velocity $\vec{v}$ and the magnetic field $\vec{B}$. The magnetic field is perpendicular to the velocity, so $\vec{v}\times\vec{B}$ has magnitude vB and the magnitude of the electric field is given by $E = vB$. Since the particle has charge e and is accelerated through a potential difference V, $mv^2/2 = eV$ and $v = \sqrt{2eV/m}$. Thus,

$$E = B\sqrt{\frac{2eV}{m}} = (1.2\text{ T})\sqrt{\frac{2\left(1.60\times10^{-19}\text{C}\right)\left(10\times10^{3}\text{V}\right)}{\left(9.99\times10^{-27}\text{kg}\right)}} = 6.8\times10^{5}\text{ V/m}.$$

Problem 28-17

(a) Using Eq. 28-16, we obtain

$$v = \frac{rqB}{m_\alpha} = \frac{2eB}{4.00\,\text{u}} = \frac{2\left(4.50\times10^{-2}\ \text{m}\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(1.20\,\text{T}\right)}{\left(4.00\,\text{u}\right)\left(1.66\times10^{-27}\ \text{kg/u}\right)} = 2.60\times10^{6}\ \text{m/s}\ .$$

(b) $T = 2\pi r/v = 2\pi(4.50 \times 10^{-2}\text{ m})/(2.60 \times 10^{6}\text{ m/s}) = 1.09 \times 10^{-7}$ s.

(c) The kinetic energy of the alpha particle is

$$K = \frac{1}{2}m_\alpha v^2 = \frac{\left(4.00\,\text{u}\right)\left(1.66\times10^{-27}\ \text{kg/u}\right)\left(2.60\times10^{6}\ \text{m/s}\right)^2}{2\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 1.40\times10^{5}\ \text{eV}\ .$$

(d) $\Delta V = K/q = 1.40 \times 10^{5}\text{ eV}/2e = 7.00 \times 10^{4}$ V.

Problem 28-33

(a) If v is the speed of the positron then $v \sin \phi$ is the component of its velocity in the plane that is perpendicular to the magnetic field. Here ϕ is the angle between the velocity and the field (89°). Newton's second law yields $eBv \sin \phi = m_e(v \sin \phi)^2/r$, where r is the radius of the orbit. Thus $r = (m_e v/eB) \sin \phi$. The period is given by

$$T = \frac{2\pi r}{v\sin\phi} = \frac{2\pi m_e}{eB} = \frac{2\pi\left(9.11\times10^{-31}\,\text{kg}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.100\text{T}\right)} = 3.58\times10^{-10}\,\text{s}.$$

The equation for r is substituted to obtain the second expression for T.

(b) The pitch is the distance traveled along the line of the magnetic field in a time interval of one period. Thus $p = vT \cos \phi$. We use the kinetic energy to find the speed: $K = \frac{1}{2}m_e v^2$ means

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\left(2.00\times10^{3}\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{9.11\times10^{-31}\,\text{kg}}} = 2.65\times10^{7}\ \text{m/s}\ .$$

Thus,

$$p = \left(2.65\times10^{7}\ \text{m/s}\right)\left(3.58\times10^{-10}\ \text{s}\right)\cos 89° = 1.66\times10^{-4}\ \text{m}\ .$$

(c) The orbit radius is

$$R = \frac{m_e v\sin\phi}{eB} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(2.65\times10^{7}\ \text{m/s}\right)\sin 89°}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.100\ \text{T}\right)} = 1.51\times10^{-3}\ \text{m}\ .$$

Problem 28-36

(a) The magnitude of the field required to achieve resonance is

$$B=\frac{2\pi fm_p}{q}=\frac{2\pi(12.0\times10^6\ \text{Hz})\left(1.67\times10^{-27}\,\text{kg}\right)}{1.60\times10^{-19}\,\text{C}}=0.787\,\text{T}.$$

(b) The kinetic energy is given by

$$\begin{aligned}K&=\tfrac{1}{2}mv^2=\frac{1}{2}m\left(2\pi Rf\right)^2=\frac{1}{2}\left(1.67\times10^{-27}\,\text{kg}\right)4\pi^2(0.530\ \text{m})^2(12.0\times10^6\ \text{Hz})^2\\&=1.33\times10^{-12}\,\text{J}=8.34\times10^6\,\text{eV}.\end{aligned}$$

(c) The required frequency is

$$f=\frac{qB}{2\pi m_p}=\frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.57\,\text{T}\right)}{2\pi\left(1.67\times10^{-27}\,\text{kg}\right)}=2.39\times10^7\,\text{Hz}.$$

(d) The kinetic energy is given by

$$\begin{aligned}K&=\tfrac{1}{2}mv^2=\frac{1}{2}m\left(2\pi Rf\right)^2=\frac{1}{2}\left(1.67\times10^{-27}\,\text{kg}\right)4\pi^2(0.530\ \text{m})^2(2.39\times10^7\ \text{Hz})^2\\&=5.3069\times10^{-12}\,\text{J}=3.32\times10^7\,\text{eV}.\end{aligned}$$

Problem 28-47

(a) The magnetic force must push horizontally on the rod to overcome the force of friction, but it can be oriented so that it also pulls up on the rod and thereby reduces both the normal force and the force of friction. The forces acting on the rod are: $\vec{F}$, the force of the magnetic field; *mg*, the magnitude of the (downward) force of gravity; $\vec{F}_N$, the normal force exerted by the stationary rails upward on the rod; and $\vec{f}$, the (horizontal) force of friction. For definiteness, we assume the rod is on the verge of moving eastward, which means that $\vec{f}$ points westward (and is equal to its maximum possible value $\mu_s F_N$). Thus, $\vec{F}$ has an eastward component F_x and an upward component F_y, which can be related to the components of the magnetic field once we assume a direction for the current in the rod. Thus, again for definiteness, we assume the current flows northward. Then, by the right-hand rule, a downward component (B_d) of $\vec{B}$ will produce the eastward F_x, and a westward component (B_w) will produce the upward F_y. Specifically,

$$F_x=iLB_d,\quad F_y=iLB_w.$$

Considering forces along a vertical axis, we find

$$F_N=mg-F_y=mg-iLB_w$$

so that

$$f = f_{s,\max} = \mu_s\left(mg - iLB_w\right).$$

It is on the verge of motion, so we set the horizontal acceleration to zero:

$$F_x - f = 0 \;\Rightarrow\; iLB_d = \mu_s\left(mg - iLB_w\right).$$

The angle of the field components is adjustable, and we can minimize with respect to it. Defining the angle by $B_w = B\sin\theta$ and $B_d = B\cos\theta$ (which means θ is being measured from a vertical axis) and writing the above expression in these terms, we obtain

$$iLB\cos\theta = \mu_s\left(mg - iLB\sin\theta\right) \;\Rightarrow\; B = \frac{\mu_s mg}{iL\left(\cos\theta + \mu_s\sin\theta\right)}$$

which we differentiate (with respect to θ) and set the result equal to zero. This provides a determination of the angle:

$$\theta = \tan^{-1}\left(\mu_s\right) = \tan^{-1}\left(0.60\right) = 31°.$$

Consequently,

$$B_{\min} = \frac{0.60\left(1.0\,\text{kg}\right)\left(9.8\,\text{m/s}^2\right)}{\left(50\,\text{A}\right)\left(1.0\,\text{m}\right)\left(\cos 31° + 0.60\sin 31°\right)} = 0.10\,\text{T}.$$

(b) As shown above, the angle is $\theta = \tan^{-1}\left(\mu_s\right) = \tan^{-1}\left(0.60\right) = 31°$.

Problem 28-49

The applied field has two components: $B_x > 0$ and $B_z > 0$. Considering each straight segment of the rectangular coil, we note that Eq. 28-26 produces a nonzero force only for the component of $\vec{B}$ that is perpendicular to that segment; we also note that the equation is effectively multiplied by $N = 20$ due to the fact that this is a 20-turn coil. Since we wish to compute the torque about the hinge line, we can ignore the force acting on the straight segment of the coil that lies along the y axis (forces acting at the axis of rotation produce no torque about that axis). The top and bottom straight segments experience forces due to Eq. 28-26 (caused by the B_z component), but these forces are (by the right-hand rule) in the $\pm y$ directions and are thus unable to produce a torque about the y axis. Consequently, the torque derives completely from the force exerted on the straight segment located at $x = 0.050$ m, which has length $L = 0.10$ m and is shown in Figure 28-44 carrying current in the $-y$ direction. Now, the B_z component will produce a force on this straight segment which points in the $-x$ direction (back towards the hinge) and thus will exert no torque about the hinge. However, the B_x component (which is equal to $B\cos\theta$ where $B = 0.50$ T and $\theta = 30°$) produces a force equal to $NiLB_x$ that points (by the right-hand rule) in the $+z$ direction. Since the action of this force is perpendicular to the plane of the coil, and is located a distance x away from the hinge, then the torque has magnitude

$$\begin{aligned}\tau &= \left(NiLB_x\right)\left(x\right) = NiLxB\cos\theta = \left(20\right)\left(0.10\text{ A}\right)\left(0.10\text{ m}\right)\left(0.050\text{ m}\right)\left(0.50\text{ T}\right)\cos 30° \\ &= 0.0043\text{ N}\cdot\text{m}.\end{aligned}$$

Since $\vec{\tau} = \vec{r} \times \vec{F}$, the direction of the torque is $-y$. In unit-vector notation, the torque is $\vec{\tau} = (-4.3\times10^{-3}\ \mathrm{N\cdot m})\hat{\mathrm{j}}$.

An alternative way to do this problem is through the use of Eq. 28-37. We do not show those details here, but note that the magnetic moment vector (a necessary part of Eq. 28-37) has magnitude

$$|\vec{\mu}| = NiA = (20)(0.10\,\mathrm{A})(0.0050\,\mathrm{m}^2)$$

and points in the $-z$ direction. At this point, Eq. 3-30 may be used to obtain the result for the torque vector.

Problem 28-51

We use Eq. 28-37 where $\vec{\mu}$ is the magnetic dipole moment of the wire loop and $\vec{B}$ is the magnetic field, as well as Newton's second law. Since the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity mg, acting downward from the center of mass, the normal force of the incline F_N, acting perpendicularly to the incline through the center of mass, and the force of friction f, acting up the incline at the point of contact. We take the x axis to be positive down the incline. Then the x component of Newton's second law for the center of mass yields

$$mg\sin\theta - f = ma.$$

For purposes of calculating the torque, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $\mu B\sin\theta$, and the force of friction produces a torque with magnitude fr, where r is the radius of the cylinder. The first tends to produce an angular acceleration in the counterclockwise direction, and the second tends to produce an angular acceleration in the clockwise direction. Newton's second law for rotation about the center of the cylinder, $\tau = I\alpha$, gives

$$fr - \mu B\sin\theta = I\alpha.$$

Since we want the current that holds the cylinder in place, we set $a = 0$ and $\alpha = 0$, and use one equation to eliminate f from the other. The result is $mgr = \mu B$. The loop is rectangular with two sides of length L and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is $\mu = NiA = Ni(2rL)$. Thus, $mgr = 2NirLB$ and

$$i = \frac{mg}{2NLB} = \frac{(0.250\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{2(10.0)(0.100\,\mathrm{m})(0.500\,\mathrm{T})} = 2.45\,\mathrm{A}.$$

Problem 28-60

Let a = 30.0 cm, b = 20.0 cm, and c = 10.0 cm. From the given hint, we write

$$\vec{\mu} = \vec{\mu}_1 + \vec{\mu}_2 = iab(-\hat{\mathrm{k}}) + iac(\hat{\mathrm{j}}) = ia(c\hat{\mathrm{j}} - b\hat{\mathrm{k}}) = (5.00\,\mathrm{A})(0.300\,\mathrm{m})\left[(0.100\,\mathrm{m})\hat{\mathrm{j}} - (0.200\,\mathrm{m})\hat{\mathrm{k}}\right]$$
$$= (0.150\hat{\mathrm{j}} - 0.300\hat{\mathrm{k}})\,\mathrm{A\cdot m^2}.$$

Problem 28-74

Letting $B_x = B_y = B_1$ and $B_z = B_2$ and using Eq. 28-2 ($\vec{F} = q\vec{v} \times \vec{B}$) and Eq. 3-30, we obtain (with SI units understood)

$$4\hat{\mathrm{i}} - 20\hat{\mathrm{j}} + 12\hat{\mathrm{k}} = 2\Big((4B_2 - 6B_1)\,\hat{\mathrm{i}} + (6B_1 - 2B_2)\,\hat{\mathrm{j}} + (2B_1 - 4B_1)\,\hat{\mathrm{k}} \Big).$$

Equating like components, we find $B_1 = -3$ and $B_2 = -4$. In summary,

$$\vec{B} = \left(-3.0\hat{\mathrm{i}} - 3.0\hat{\mathrm{j}} - 4.0\hat{\mathrm{k}} \right)\mathrm{T}.$$