Physics 4B

Solutions to Chapter 29 HW

Chapter 29: Questions: 2, 4, 10
Problems: 8, 13, 15, 36, 41, 48, 55, 57, 90

Question 29-2
1, then 3 and 4 tie, then 2 (zero)

Question 29-4
(a) into;(b) greater

Question 29-10
d, then *a* and *e* tie, then *b*, *c*

Problem 29-8
(a) Recalling the *straight sections* discussion in Sample Problem — "Magnetic field at the center of a circular arc of current," we see that the current in segments *AH* and *JD* do not contribute to the field at point *C*. Using Eq. 29-9 (with $\phi = \pi$) and the right-hand rule, we find that the current in the semicircular arc *H J* contributes $\mu_0 i/4R_1$ (into the page) to the field at *C*. Also, arc *D A* contributes $\mu_0 i/4R_2$ (out of the page) to the field there. Thus, the net field at *C* is

$$B = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.281\text{A})}{4}\left(\frac{1}{0.0315\text{m}} - \frac{1}{0.0780\text{m}}\right) = 1.67\times10^{-6}\,\text{T}.$$

(b) The direction of the field is into the page.

Problem 29-13
Our *x* axis is along the wire with the origin at the midpoint. The current flows in the positive *x* direction. All segments of the wire produce magnetic fields at P_1 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_1 is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}dx$$

where θ (the angle between the segment and a line drawn from the segment to P_1) and *r* (the length of that line) are functions of *x*. Replacing *r* with $\sqrt{x^2+R^2}$ and sin θ with $R/r = R\big/\sqrt{x^2+R^2}$, we integrate from $x = -L/2$ to $x = L/2$. The total field is

$$B=\frac{\mu_0 iR}{4\pi}\int_{-L/2}^{L/2}\frac{dx}{\left(x^2+R^2\right)^{3/2}}=\frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{\left(x^2+R^2\right)^{1/2}}\Bigg|_{-L/2}^{L/2}=\frac{\mu_0 i}{2\pi R}\frac{L}{\sqrt{L^2+4R^2}}$$

$$=\frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(0.0582\ \mathrm{A}\right)}{2\pi\left(0.131\,\mathrm{m}\right)}\frac{0.180\mathrm{m}}{\sqrt{(0.180\mathrm{m})^2+4(0.131\mathrm{m})^2}}=5.03\times10^{-8}\ \mathrm{T}.$$

Problem 29-15

(a) As discussed in Sample Problem — "Magnetic field at the center of a circular arc of current," the radial segments do not contribute to $\vec{B}_P$ and the arc segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\mathrm{k}}$ designates the direction "out of the page" then

$$\vec{B}=\frac{\mu_0\left(0.40\,\mathrm{A}\right)\left(\pi\,\mathrm{rad}\right)}{4\pi\left(0.050\,\mathrm{m}\right)}\hat{\mathrm{k}}-\frac{\mu_0\left(0.80\,\mathrm{A}\right)\left(2\pi/3\,\mathrm{rad}\right)}{4\pi\left(0.040\,\mathrm{m}\right)}\hat{\mathrm{k}}=-(1.7\times10^{-6}\ \mathrm{T})\hat{\mathrm{k}}$$

or $|\vec{B}|=1.7\times10^{-6}\,\mathrm{T}$.

(b) The direction is $-\hat{\mathrm{k}}$, or into the page.

(c) If the direction of i_1 is reversed, we then have

$$\vec{B}=-\frac{\mu_0\left(0.40\,\mathrm{A}\right)\left(\pi\,\mathrm{rad}\right)}{4\pi\left(0.050\,\mathrm{m}\right)}\hat{\mathrm{k}}-\frac{\mu_0\left(0.80\,\mathrm{A}\right)\left(2\pi/3\,\mathrm{rad}\right)}{4\pi\left(0.040\,\mathrm{m}\right)}\hat{\mathrm{k}}=-(6.7\times10^{-6}\ \mathrm{T})\hat{\mathrm{k}}$$

or $|\vec{B}|=6.7\times10^{-6}\mathrm{T}$.

(d) The direction is $-\hat{\mathrm{k}}$, or into the page.

Problem 29-36

We label these wires 1 through 5, left to right, and use Eq. 29-13. Then,

(a) The magnetic force on wire 1 is

$$\vec{F}_1=\frac{\mu_0 i^2 l}{2\pi}\left(\frac{1}{d}+\frac{1}{2d}+\frac{1}{3d}+\frac{1}{4d}\right)\hat{\mathrm{j}}=\frac{25\mu_0 i^2 l}{24\pi d}\hat{\mathrm{j}}=\frac{25\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(3.00\mathrm{A}\right)^2(10.0\mathrm{m})}{24\pi\left(8.00\times10^{-2}\mathrm{m}\right)}\hat{\mathrm{j}}$$

$$=(4.69\times10^{-4}\,\mathrm{N})\,\hat{\mathrm{j}}.$$

(b) Similarly, for wire 2, we have

$$\vec{F}_2=\frac{\mu_0 i^2 l}{2\pi}\left(\frac{1}{2d}+\frac{1}{3d}\right)\hat{\mathrm{j}}=\frac{5\mu_0 i^2 l}{12\pi d}\hat{\mathrm{j}}=(1.88\times10^{-4}\mathrm{N})\,\hat{\mathrm{j}}.$$

(c) $F_3 = 0$ (because of symmetry).

(d) $\vec{F}_4 = -\vec{F}_2 = (-1.88\times10^{-4}\,\mathrm{N})\hat{\mathrm{j}}$, and

(e) $\vec{F}_5 = -\vec{F}_1 = -(4.69\times10^{-4}\,\mathrm{N})\hat{\mathrm{j}}$.

Problem 29-41

The magnitudes of the forces on the sides of the rectangle that are parallel to the long straight wire (with $i_1 = 30.0$ A) are computed using Eq. 29-13, but the force on each of the sides lying perpendicular to it (along our y axis, with the origin at the top wire and $+y$ downward) would be figured by integrating as follows:

$$F_{\perp\,\text{sides}} = \int_a^{a+b} \frac{i_2\mu_0 i_1}{2\pi y}\,dy.$$

Fortunately, these forces on the two perpendicular sides of length b cancel out. For the remaining two (parallel) sides of length L, we obtain

$$F = \frac{\mu_0 i_1 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a+d}\right) = \frac{\mu_0 i_1 i_2 b}{2\pi a(a+b)}$$
$$= \frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(30.0\mathrm{A}\right)\left(20.0\mathrm{A}\right)\left(8.00\,\mathrm{cm}\right)\left(300\times10^{-2}\,\mathrm{m}\right)}{2\pi\left(1.00\,\mathrm{cm} + 8.00\mathrm{cm}\right)} = 3.20\times10^{-3}\,\mathrm{N},$$

and $\vec{F}$ points toward the wire, or $+\hat{\mathrm{j}}$. That is, $\vec{F} = (3.20\times10^{-3}\,\mathrm{N})\hat{\mathrm{j}}$ in unit-vector notation.

Problem 29-48

(a) The field at the center of the pipe (point C) is due to the wire alone, with a magnitude of

$$B_C = \frac{\mu_0 i_{\text{wire}}}{2\pi(3R)} = \frac{\mu_0 i_{\text{wire}}}{6\pi R}.$$

For the wire we have $B_{P,\,\text{wire}} > B_{C,\,\text{wire}}$. Thus, for $B_P = B_C = B_{C,\,\text{wire}}$, i_{wire} must be into the page:

$$B_P = B_{P,\text{wire}} - B_{P,\text{pipe}} = \frac{\mu_0 i_{\text{wire}}}{2\pi R} - \frac{\mu_0 i}{2\pi(2R)}.$$

Setting $B_C = -B_P$ we obtain $i_{\text{wire}} = 3i/8 = 3(8.00\times10^{-3}\,\mathrm{A})/8 = 3.00\times10^{-3}\,\mathrm{A}$.

(b) The direction is into the page.

Problem 29-55

(a) We denote the $\vec{B}$ fields at point P on the axis due to the solenoid and the wire as $\vec{B}_s$ and $\vec{B}_w$, respectively. Since $\vec{B}_s$ is along the axis of the solenoid and $\vec{B}_w$ is perpendicular to it, $\vec{B}_s \perp \vec{B}_w$. For the net field $\vec{B}$ to be at 45° with the axis we then must have $B_s = B_w$. Thus,

$$B_s = \mu_0 i_s n = B_w = \frac{\mu_0 i_w}{2\pi d} ,$$

which gives the separation d to point P on the axis:

$$d = \frac{i_w}{2\pi i_s n} = \frac{6.00\,\text{A}}{2\pi\left(20.0\times10^{-3}\,\text{A}\right)\left(10\,\text{turns/cm}\right)} = 4.77\,\text{cm}.$$

(b) The magnetic field strength is

$$B = \sqrt{2}B_s = \sqrt{2}\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(20.0\times10^{-3}\,\text{A}\right)\left(10\ \text{turns}/0.0100\ \text{m}\right) = 3.55\times10^{-5}\,\text{T}.$$

Problem 29-57

(a) The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. We use $A = \pi R^2$, where R is the radius. Thus,

$$\mu = Ni\pi R^2 = (300)(4.0\,\text{A})\pi(0.025\,\text{m})^2 = 2.4\,\text{A}\cdot\text{m}^2 .$$

(b) The magnetic field on the axis of a magnetic dipole, a distance z away, is given by Eq. 29-27:

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3} .$$

We solve for z:

$$z = \left(\frac{\mu_0}{2\pi}\frac{\mu}{B}\right)^{1/3} = \left(\frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(2.36\,\text{A}\cdot\text{m}^2\right)}{2\pi\left(5.0\times10^{-6}\,\text{T}\right)}\right)^{1/3} = 46\,\text{cm} .$$

Problem 29-90

(a) The magnitude of the magnetic field on the axis of a circular loop, a distance z from the loop center, is given by Eq. 29-26:

$$B = \frac{N\mu_0 i R^2}{2(R^2+z^2)^{3/2}},$$

where R is the radius of the loop, N is the number of turns, and i is the current. Both of the loops in the problem have the same radius, the same number of turns, and carry the same current. The currents are in the same sense, and the fields they produce are in the same direction in the region between them. We place the origin at the center of the left-hand loop and let x be the coordinate of a point on the axis between the loops. To calculate the field of the left-hand loop, we set $z = x$ in the equation above. The chosen point on the axis is a distance $s - x$ from the center of the right-hand loop. To calculate the field it produces, we put $z = s - x$ in the equation above. The total field at the point is therefore

$$B = \frac{N\mu_0 iR^2}{2}\left[\frac{1}{(R^2+x^2)^{3/2}} + \frac{1}{(R^2+x^2-2sx+s^2)^{3/2}}\right].$$

Its derivative with respect to x is

$$\frac{dB}{dx} = -\frac{N\mu_0 iR^2}{2}\left[\frac{3x}{(R^2+x^2)^{5/2}} + \frac{3(x-s)}{(R^2+x^2-2sx+s^2)^{5/2}}\right].$$

When this is evaluated for $x = s/2$ (the midpoint between the loops) the result is

$$\left.\frac{dB}{dx}\right|_{s/2} = -\frac{N\mu_0 iR^2}{2}\left[\frac{3s/2}{(R^2+s^2/4)^{5/2}} - \frac{3s/2}{(R^2+s^2/4-s^2+s^2)^{5/2}}\right] = 0$$

independent of the value of s.

(b) The second derivative is

$$\frac{d^2B}{dx^2} = \frac{N\mu_0 iR^2}{2}\left[-\frac{3}{(R^2+x^2)^{5/2}} + \frac{15x^2}{(R^2+x^2)^{7/2}}\right.$$
$$\left. - \frac{3}{(R^2+x^2-2sx+s^2)^{5/2}} + \frac{15(x-s)^2}{(R^2+x^2-2sx+s^2)^{7/2}}\right].$$

At $x = s/2$,

$$\left.\frac{d^2B}{dx^2}\right|_{s/2} = \frac{N\mu_0 iR^2}{2}\left[-\frac{6}{(R^2+s^2/4)^{5/2}} + \frac{30s^2/4}{(R^2+s^2/4)^{7/2}}\right]$$
$$= \frac{N\mu_0 R^2}{2}\left[\frac{-6(R^2+s^2/4)+30s^2/4}{(R^2+s^2/4)^{7/2}}\right] = 3N\mu_0 iR^2\,\frac{s^2-R^2}{(R^2+s^2/4)^{7/2}}.$$

Clearly, this is zero if $s = R$.