Physics 4B

Solutions to Chapter 30 HW

Chapter 30: Questions: 2, 4, 10
Problems: 2, 15, 19, 27, 33, 41, 45, 54, 65

Question 30-2
1 and 3 tie (clockwise), then 2 and 5 tie (zero), then 4 and 6 tie (counterclockwise)

Question 30-4
(a) into;(b) counterclockwise;(c) larger

Question 30-10
c, *b*, *a*

Problem 30-2
Using Faraday's law, the induced emf is

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d\left(BA\right)}{dt} = -B\frac{dA}{dt} = -B\frac{d\left(\pi r^2\right)}{dt} = -2\pi rB\frac{dr}{dt}\\ &= -2\pi\left(0.12\text{m}\right)\left(0.800\text{T}\right)\left(-0.750\text{m/s}\right)\\ &= 0.452\text{V}.\end{aligned}$$

Problem 30-15
(a) Let L be the length of a side of the square circuit. Then the magnetic flux through the circuit is $\Phi_B = L^2B/2$, and the induced emf is

$$\varepsilon_i = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt}.$$

Now $B = 0.042 - 0.870t$ and $dB/dt = -0.870$ T/s. Thus,

$$\varepsilon_i = \frac{(2.00\text{ m})^2}{2}(0.870\text{ T/s}) = 1.74\text{ V}.$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is

$$\varepsilon + \varepsilon_i = 20.0\text{ V} + 1.74\text{ V} = 21.7\text{ V}.$$

(b) The current is in the sense of the total emf (counterclockwise).

Problem 30-19

First we write $\Phi_B = BA \cos \theta$. We note that the angular position θ of the rotating coil is measured from some reference line or plane, and we are implicitly making such a choice by writing the magnetic flux as $BA \cos \theta$ (as opposed to, say, $BA \sin \theta$). Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ if θ is understood to be in radians (here, $\omega = 2\pi f$ is the angular velocity of the coil in radians per second, and f = 1000 rev/min ≈ 16.7 rev/s is the frequency). Since the area of the rectangular coil is A = (0.500 m) × (0.300 m) = 0.150 m^2, Faraday's law leads to

$$\varepsilon = -N\frac{d\left(BA\cos\theta\right)}{dt} = -NBA\frac{d\cos\left(2\pi ft\right)}{dt} = NBA2\pi f\,\sin\left(2\pi ft\right)$$

which means it has a voltage amplitude of

$$\varepsilon_{\max} = 2\pi\, fNAB = 2\pi\left(16.7\,\text{rev/s}\right)\left(100\,\text{turns}\right)\left(0.15\,\text{m}^2\right)\left(3.5\,\text{T}\right) = 5.50\times10^3\ \text{V}\ .$$

Problem 30-27

(a) Consider a (thin) strip of area of height dy and width $\ell = 0.020\,\text{m}$. The strip is located at some $0 < y < \ell$. The element of flux through the strip is

$$d\Phi_B = BdA = \left(4t^2 y\right)\left(\ell dy\right)$$

where SI units (and 2 significant figures) are understood. To find the total flux through the square loop, we integrate:

$$\Phi_B = \int d\Phi_B = \int_0^{\ell}\left(4t^2 y\ell\right)dy = 2t^2\ell^3\ .$$

Thus, Faraday's law yields

$$\left|\varepsilon\right| = \left|\frac{d\Phi_B}{dt}\right| = 4t\ell^3\ .$$

At t = 2.5 s, the magnitude of the induced emf is 8.0 × 10^{-5} V.

(b) Its "direction" (or "sense'') is clockwise, by Lenz's law.

Problem 30-33

(a) Letting x be the distance from the right end of the rails to the rod, we find an expression for the magnetic flux through the area enclosed by the rod and rails. By Eq. 29-17, the field is $B = \mu_0 i/2\pi r$, where r is the distance from the long straight wire. We consider an infinitesimal horizontal strip of length x and width dr, parallel to the wire and a distance r from it; it has area $A = x\,dr$ and the flux is

$$d\Phi_B = BdA = \frac{\mu_0 i}{2\pi r}\,xdr\ .$$

By Eq. 30-1, the total flux through the area enclosed by the rod and rails is

$$\Phi_B = \frac{\mu_0 i x}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i x}{2\pi}\ln\left(\frac{a+L}{a}\right).$$

According to Faraday's law the emf induced in the loop is

$$\begin{aligned}\varepsilon &= \frac{d\Phi_B}{dt} = \frac{\mu_0 i}{2\pi}\frac{dx}{dt}\ln\left(\frac{a+L}{a}\right) = \frac{\mu_0 i v}{2\pi}\ln\left(\frac{a+L}{a}\right)\\ &= \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(100\,\text{A})(5.00\,\text{m/s})}{2\pi}\ln\left(\frac{1.00\,\text{cm}+10.0\,\text{cm}}{1.00\,\text{cm}}\right) = 2.40\times10^{-4}\,\text{V}.\end{aligned}$$

(b) By Ohm's law, the induced current is

$$i_\ell = \varepsilon / R = (2.40\times10^{-4}\,\text{V})/(0.400\,\Omega) = 6.00\times10^{-4}\,\text{A}.$$

Since the flux is increasing, the magnetic field produced by the induced current must be into the page in the region enclosed by the rod and rails. This means the current is clockwise.

(c) Thermal energy is being generated at the rate

$$P = i_\ell^2 R = (6.00\times10^{-4}\,\text{A})^2(0.400\,\Omega) = 1.44\times10^{-7}\,\text{W}.$$

(d) Since the rod moves with constant velocity, the net force on it is zero. The force of the external agent must have the same magnitude as the magnetic force and must be in the opposite direction. The magnitude of the magnetic force on an infinitesimal segment of the rod, with length *dr* at a distance *r* from the long straight wire, is

$$dF_B = i_\ell B\,dr = (\mu_0 i_\ell i / 2\pi r)\,dr.$$

We integrate to find the magnitude of the total magnetic force on the rod:

$$\begin{aligned}F_B &= \frac{\mu_0 i_\ell i}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i_\ell i}{2\pi}\ln\left(\frac{a+L}{a}\right)\\ &= \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(6.00\times10^{-4}\,\text{A})(100\,\text{A})}{2\pi}\ln\left(\frac{1.00\,\text{cm}+10.0\,\text{cm}}{1.00\,\text{cm}}\right)\\ &= 2.87\times10^{-8}\,\text{N}.\end{aligned}$$

Since the field is out of the page and the current in the rod is upward in the diagram, the force associated with the magnetic field is toward the right. The external agent must therefore apply a force of 2.87×10^{-8} N, to the left.

(e) By Eq. 7-48, the external agent does work at the rate

$$P = Fv = (2.87 \times 10^{-8}\ \text{N})(5.00\ \text{m/s}) = 1.44 \times 10^{-7}\ \text{W}.$$

This is the same as the rate at which thermal energy is generated in the rod. All the energy supplied by the agent is converted to thermal energy.

Problem 30-41

(a) We interpret the question as asking for *N* multiplied by the flux through one turn:

$$\Phi_{\text{turns}} = N\Phi_B = NBA = NB\left(\pi r^2\right) = (30.0)\left(2.60\times10^{-3}\ \text{T}\right)(\pi)(0.100\ \text{m})^2 = 2.45\times10^{-3}\ \text{Wb}.$$

(b) Equation 30-33 leads to

$$L = \frac{N\Phi_B}{i} = \frac{2.45\times10^{-3}\ \text{Wb}}{3.80\ \text{A}} = 6.45\times10^{-4}\ \text{H}.$$

Problem 30-45

(a) Speaking anthropomorphically, the coil wants to fight the changes—so if it wants to push current rightward (when the current is already going rightward) then *i* must be in the process of decreasing.

(b) From Eq. 30-35 (in absolute value) we get

$$L = \left|\frac{\varepsilon}{di/dt}\right| = \frac{17\ \text{V}}{2.5\ \text{kA/s}} = 6.8\times10^{-4}\ \text{H}.$$

Problem 30-54

54. (a) The inductor prevents a fast build-up of the current through it, so immediately after the switch is closed, the current in the inductor is zero. It follows that

$$i_1 = \frac{\varepsilon}{R_1+R_2} = \frac{100\ \text{V}}{10.0\ \Omega+20.0\ \Omega} = 3.33\ \text{A}.$$

(b) $i_2 = i_1 = 3.33\ \text{A}.$

(c) After a suitably long time, the current reaches steady state. Then, the emf across the inductor is zero, and we may imagine it replaced by a wire. The current in R_3 is $i_1 - i_2$. Kirchhoff's loop rule gives

$$\varepsilon - i_1R_1 - i_2R_2 = 0$$
$$\varepsilon - i_1R_1 - (i_1 - i_2)R_3 = 0.$$

We solve these simultaneously for i_1 and i_2, and find

$$i_1=\frac{\varepsilon\left(R_2+R_3\right)}{R_1R_2+R_1R_3+R_2R_3}=\frac{(100\,\mathrm{V})(20.0\,\Omega+30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega)+(10.0\,\Omega)(30.0\,\Omega)+(20.0\,\Omega)(30.0\,\Omega)}$$
$$=4.55\,\mathrm{A}\,,$$

(d) and

$$i_2=\frac{\varepsilon R_3}{R_1R_2+R_1R_3+R_2R_3}=\frac{(100\,\mathrm{V})(30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega)+(10.0\,\Omega)(30.0\,\Omega)+(20.0\,\Omega)(30.0\,\Omega)}$$
$$=2.73\,\mathrm{A}.$$

(e) The left-hand branch is now broken. We take the current (immediately) as zero in that branch when the switch is opened (that is, $i_1 = 0$).

(f) The current in R_3 changes less rapidly because there is an inductor in its branch. In fact, immediately after the switch is opened it has the same value that it had before the switch was opened. That value is 4.55 A – 2.73 A = 1.82 A. The current in R_2 is the same but in the opposite direction as that in R_3, that is, $i_2 = -1.82$ A.

A long time later after the switch is reopened, there are no longer any sources of emf in the circuit, so all currents eventually drop to zero. Thus,

(g) $i_1 = 0$, and

(h) $i_2 = 0$.

Problem 30-65

(a) The energy delivered by the battery is the integral of Eq. 28-14 (where we use Eq. 30-41 for the current):

$$\int_0^t P_{\text{battery}}dt=\int_0^t\frac{\varepsilon^2}{R}\left(1-e^{-Rt/L}\right)dt=\frac{\varepsilon^2}{R}\left[t+\frac{L}{R}\left(e^{-Rt/L}-1\right)\right]$$
$$=\frac{(10.0\,\mathrm{V})^2}{6.70\ \Omega}\left[2.00\ \mathrm{s}+\frac{(5.50\,\mathrm{H})\left(e^{-(6.70\ \Omega)(2.00\ \mathrm{s})/5.50\ \mathrm{H}}-1\right)}{6.70\ \Omega}\right]$$
$$=18.7\ \mathrm{J}.$$

(b) The energy stored in the magnetic field is given by Eq. 30-49:

$$U_B=\frac{1}{2}Li^2(t)=\frac{1}{2}L\left(\frac{\varepsilon}{R}\right)^2\left(1-e^{-Rt/L}\right)^2=\frac{1}{2}(5.50\,\mathrm{H})\left(\frac{10.0\,\mathrm{V}}{6.70\,\Omega}\right)^2\left[1-e^{-(6.70\ \Omega)(2.00\ \mathrm{s})/5.50\ \mathrm{H}}\right]^2$$
$$=5.10\ \mathrm{J}\ .$$

(c) The difference of the previous two results gives the amount "lost" in the resistor:
18.7 J – 5.10 J = 13.6 J.