Physics 4B

Solutions to Chapter 31 HW

Chapter 31: Questions: 2, 8, 12
Exercises & Problems: 2, 23, 24, 32, 41, 44, 48, 60, 72, 83

Question 31-2
(a) less; (b) greater

Question 31-8
(a) 1 and 4; (b) 2 and 3

Question 31-12
(a) lead; (b) capacitive; (c) less

Problem 31-2
(a) We recall the fact that the period is the reciprocal of the frequency. It is helpful to refer also to Fig. 31-1. The values of *t* when plate *A* will again have maximum positive charge are multiples of the period:

$$t_A = nT = \frac{n}{f} = \frac{n}{2.00\times 10^3\ \text{Hz}} = n(5.00\ \mu\text{s}),$$

where *n* = 1, 2, 3, 4, …. The earliest time is (*n* = 1) $t_A = 5.00\mu$s.

(b) We note that it takes $t = \frac{1}{2}T$ for the charge on the other plate to reach its maximum positive value for the first time (compare steps *a* and *e* in Fig. 31-1). This is when plate *A* acquires its most negative charge. From that time onward, this situation will repeat once every period. Consequently,

$$t = \frac{1}{2}T + (n-1)T = \frac{1}{2}(2n-1)T = \frac{(2n-1)}{2f} = \frac{(2n-1)}{2(2\times 10^3\text{Hz})} = (2n-1)(2.50\mu\text{s}),$$

where *n* = 1, 2, 3, 4, …. The earliest time is (*n* = 1) $t = 2.50\mu$s.

(c) At $t = \frac{1}{4}T$, the current and the magnetic field in the inductor reach maximum values for the first time (compare steps *a* and *c* in Fig. 31-1). Later this will repeat every half-period (compare steps *c* and *g* in Fig. 31-1). Therefore,

$$t_L = \frac{T}{4} + \frac{(n-1)T}{2} = (2n-1)\frac{T}{4} = (2n-1)(1.25\mu\text{s}),$$

where n = 1, 2, 3, 4, …. The earliest time is (n = 1) $t = 1.25\,\mu$s.

Problem 31-23

(a) The total energy U is the sum of the energies in the inductor and capacitor:

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{i^2 L}{2} = \frac{\left(3.80\times10^{-6}\,\mathrm{C}\right)^2}{2\left(7.80\times10^{-6}\,\mathrm{F}\right)} + \frac{\left(9.20\times10^{-3}\,\mathrm{A}\right)^2\left(25.0\times10^{-3}\,\mathrm{H}\right)}{2} = 1.98\times10^{-6}\,\mathrm{J}.$$

(b) We solve $U = Q^2/2C$ for the maximum charge:

$$Q = \sqrt{2CU} = \sqrt{2\left(7.80\times10^{-6}\,\mathrm{F}\right)\left(1.98\times10^{-6}\,\mathrm{J}\right)} = 5.56\times10^{-6}\,\mathrm{C}.$$

(c) From $U = I^2L/2$, we find the maximum current:

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2\left(1.98\times10^{-6}\,\mathrm{J}\right)}{25.0\times10^{-3}\,\mathrm{H}}} = 1.26\times10^{-2}\,\mathrm{A}.$$

(d) If q_0 is the charge on the capacitor at time t = 0, then $q_0 = Q\cos\phi$ and

$$\phi = \cos^{-1}\left(\frac{q}{Q}\right) = \cos^{-1}\left(\frac{3.80\times10^{-6}\,\mathrm{C}}{5.56\times10^{-6}\,\mathrm{C}}\right) = \pm46.9°.$$

For ϕ = +46.9° the charge on the capacitor is decreasing, for ϕ = –46.9° it is increasing. To check this, we calculate the derivative of q with respect to time, evaluated for t = 0. We obtain $-\omega Q\sin\phi$, which we wish to be positive. Since sin(+46.9°) is positive and sin(–46.9°) is negative, the correct value for increasing charge is ϕ = –46.9°.

(e) Now we want the derivative to be negative and $\sin\phi$ to be positive. Thus, we take ϕ = +46.9°.

Problem 31-24

The charge q after N cycles is obtained by substituting $t = NT = 2\pi N/\omega'$ into Eq. 31-25:

$$\begin{aligned} q &= Qe^{-Rt/2L}\cos\left(\omega' t + \phi\right) = Qe^{-RNT/2L}\cos\left[\omega'\left(2\pi N/\omega'\right) + \phi\right] \\ &= Qe^{-RN\left(2\pi\sqrt{L/C}\right)/2L}\cos\left(2\pi N + \phi\right) \\ &= Qe^{-N\pi R\sqrt{C/L}}\cos\phi. \end{aligned}$$

We note that the initial charge (setting $N = 0$ in the above expression) is $q_0 = Q \cos \phi$, where $q_0 = 6.2\ \mu\text{C}$ is given (with 3 significant figures understood). Consequently, we write the above result as $q_N = q_0 \exp\left(-N\pi R\sqrt{C/L}\right)$.

(a) For $N = 5$, $q_5 = (6.2\,\mu\text{C})\exp\left(-5\pi(7.2\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 5.85\,\mu\text{C}$.

(b) For $N = 10$, $q_{10} = (6.2\,\mu\text{C})\exp\left(-10\pi(7.2\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 5.52\,\mu\text{C}$.

(c) For $N = 100$, $q_{100} = (6.2\,\mu\text{C})\exp\left(-100\pi(7.2\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 1.93\,\mu\text{C}$.

Problem 31-32

(a) The circuit consists of one generator across one inductor; therefore, $\varepsilon_m = V_L$. The current amplitude is

$$I = \frac{\varepsilon_m}{X_L} = \frac{\varepsilon_m}{\omega_d L} = \frac{25.0\text{ V}}{(377\text{ rad/s})(12.7\text{ H})} = 5.22\times10^{-3}\text{ A}\ .$$

(b) When the current is at a maximum, its derivative is zero. Thus, Eq. 30-35 gives $\varepsilon_L = 0$ at that instant. Stated another way, since $\varepsilon(t)$ and $i(t)$ have a 90° phase difference, then $\varepsilon(t)$ must be zero when $i(t) = I$. The fact that $\phi = 90° = \pi/2$ rad is used in part (c).

(c) Consider Eq. 31-28 with $\varepsilon = -\varepsilon_m/2$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that ε is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 31-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [n = integer]. Consequently, Eq. 31-29 yields (for all values of n)

$$i = I\sin\left(2n\pi - \frac{5\pi}{6} - \frac{\pi}{2}\right) = (5.22\times10^{-3}\text{A})\left(\frac{\sqrt{3}}{2}\right) = 4.51\times10^{-3}\text{ A}\ .$$

Problem 31-41

(a) The capacitive reactance is

$$X_C = \frac{1}{\omega_d C} = \frac{1}{2\pi f_d C} = \frac{1}{2\pi(60.0\text{ Hz})(70.0\times10^{-6}\text{F})} = 37.9\,\Omega\ .$$

The inductive reactance 86.7 Ω is unchanged. The new impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\Omega)^2 + (37.9\Omega - 86.7\Omega)^2} = 206\Omega.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 37.9\,\Omega}{200\,\Omega}\right) = 13.7°.$$

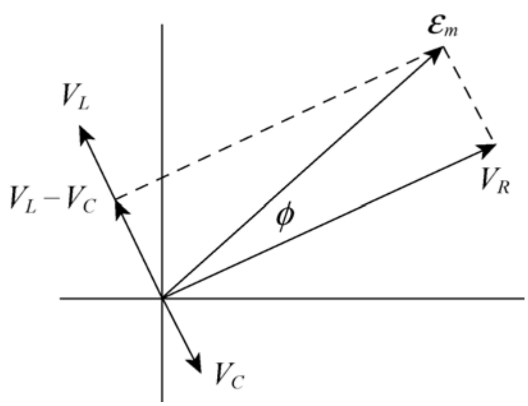
(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{36.0\text{ V}}{206\,\Omega} = 0.175\,\text{A}.$$

(d) We first find the voltage amplitudes across the circuit elements:

$$\begin{aligned} V_R &= IR = (0.175\text{ A})(200\,\Omega) = 35.0\text{ V} \\ V_L &= IX_L = (0.175\text{ A})(86.7\,\Omega) = 15.2\text{ V} \\ V_C &= IX_C = (0.175\text{ A})(37.9\,\Omega) = 6.62\,\text{V} \end{aligned}$$

Note that $X_L > X_C$, so that ε_m leads *I*. The phasor diagram is drawn to scale below.



Problem 31-44

(a) The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(400\text{ Hz})(24.0\times 10^{-6}\text{F})} = 16.6\ \Omega\,.$$

(b) The impedance is

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (2\pi fL - X_C)^2} \\ &= \sqrt{(220\,\Omega)^2 + [2\pi(400\text{ Hz})(150\times 10^{-3}\text{ H}) - 16.6\ \Omega]^2} = 422\ \Omega\,. \end{aligned}$$

(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{220\,\text{V}}{422\,\Omega} = 0.521\text{ A}\,.$$

(d) Now $X_C \propto C_{\text{eq}}^{-1}$. Thus, X_C increases as C_{eq} decreases.

(e) Now $C_{\text{eq}} = C/2$, and the new impedance is

$$Z = \sqrt{(220\ \Omega)^2 + [2\pi(400\ \text{Hz})(150\times10^{-3}\ \text{H}) - 2(16.6\ \Omega)]^2} = 408\ \Omega < 422\ \Omega\ .$$

Therefore, the impedance decreases.

(f) Since $I \propto Z^{-1}$, it increases.

Problem 31-48

(a) With both switches closed (which effectively removes the resistor from the circuit), the impedance is just equal to the (net) reactance and is equal to

$$X_{\text{net}} = (12\ \text{V})/(0.447\ \text{A}) = 26.85\ \Omega.$$

With switch 1 closed but switch 2 open, we have the same (net) reactance as just discussed, but now the resistor is part of the circuit; using Eq. 31-65 we find

$$R = \frac{X_{\text{net}}}{\tan\phi} = \frac{26.85\ \Omega}{\tan 15°} = 100\ \Omega\ .$$

(b) For the first situation described in the problem (both switches open) we can reverse our reasoning of part (a) and find

$$X_{\text{net first}} = R\tan\phi' = (100\ \Omega)\tan(-30.9°) = -59.96\ \Omega.$$

We observe that the effect of switch 1 implies

$$X_C = X_{\text{net}} - X_{\text{net first}} = 26.85\ \Omega - (-59.96\ \Omega) = 86.81\ \Omega.$$

Then Eq. 31-39 leads to $C = 1/\omega X_C = 30.6\ \mu\text{F}$.

(c) Since $X_{\text{net}} = X_L - X_C$, then we find $L = X_L/\omega = 301$ mH .

Problem 31-60

The current in the circuit satisfies $i(t) = I \sin(\omega_d t - \phi)$, where

$$\begin{aligned} I &= \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} \\ &= \frac{45.0\text{ V}}{\sqrt{(16.0\,\Omega)^2 + \{(3000\text{ rad/s})(9.20\text{ mH}) - 1/[(3000\text{ rad/s})(31.2\,\mu\text{F})]\}^2}} \\ &= 1.93\text{ A} \end{aligned}$$

and

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega_d L - 1/\omega_d C}{R}\right) \\ &= \tan^{-1}\left[\frac{(3000\text{ rad/s})(9.20\text{ mH})}{16.0\,\Omega} - \frac{1}{(3000\text{ rad/s})(16.0\,\Omega)(31.2\,\mu\text{F})}\right] \\ &= 46.5°. \end{aligned}$$

(a) The power supplied by the generator is

$$\begin{aligned} P_g &= i(t)\varepsilon(t) = I \sin(\omega_d t - \phi)\,\varepsilon_m \sin \omega_d t \\ &= (1.93\text{ A})(45.0\text{ V}) \sin[(3000\text{ rad/s})(0.442\text{ ms})] \sin[(3000\text{ rad/s})(0.442\text{ ms}) - 46.5°] \\ &= 41.4\text{ W}. \end{aligned}$$

(b) With

$$v_c(t) = V_c \sin(\omega_d t - \phi - \pi/2) = -V_c \cos(\omega_d t - \phi)$$

where $V_c = I/\omega_d C$, the rate at which the energy in the capacitor changes is

$$\begin{aligned} P_c &= \frac{d}{dt}\left(\frac{q^2}{2C}\right) = i\frac{q}{C} = iv_c \\ &= -I \sin(\omega_d t - \phi)\left(\frac{I}{\omega_d C}\right)\cos(\omega_d t - \phi) = -\frac{I^2}{2\omega_d C}\sin[2(\omega_d t - \phi)] \\ &= -\frac{(1.93\text{ A})^2}{2(3000\text{ rad/s})(31.2\times10^{-6}\text{ F})}\sin[2(3000\text{ rad/s})(0.442\text{ ms}) - 2(46.5°)] \\ &= -17.0\text{ W}. \end{aligned}$$

(c) The rate at which the energy in the inductor changes is

$$P_L = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt} = LI\sin(\omega_d t-\phi)\frac{d}{dt}\left[I\sin(\omega_d t-\phi)\right] = \frac{1}{2}\omega_d LI^2\sin\left[2(\omega_d t-\phi)\right]$$
$$= \frac{1}{2}(3000\,\text{rad/s})(1.93\,\text{A})^2(9.20\,\text{mH})\sin\left[2(3000\,\text{rad/s})(0.442\,\text{ms})-2(46.5°)\right]$$
$$= 44.1\,\text{W}.$$

(d) The rate at which energy is being dissipated by the resistor is

$$P_R = i^2R = I^2R\sin^2(\omega_d t-\phi) = (1.93\,\text{A})^2(16.0\,\Omega)\sin^2\left[(3000\,\text{rad/s})(0.442\,\text{ms})-46.5°\right]$$
$$= 14.4\,\text{W}.$$

(e) Equal. $P_L + P_R + P_c = 44.1\,\text{W} - 17.0\,\text{W} + 14.4\,\text{W} = 41.5\,\text{W} = P_g$.

Problem 31-72

(a) From Eq. 31-65, we have

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{V_L - (V_L/1.50)}{(V_L/2.00)}\right)$$

which becomes $\tan^{-1}(2/3) = 33.7°$ or 0.588 rad.

(b) Since $\phi > 0$, it is inductive ($X_L > X_C$).

(c) We have $V_R = IR = 9.98$ V, so that $V_L = 2.00V_R = 20.0$ V and $V_C = V_L/1.50 = 13.3$ V. Therefore, from Eq. 31-60, we have

$$\varepsilon_m = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(9.98\text{ V})^2 + (20.0\text{ V} - 13.3\text{ V})^2} = 12.0\text{ V}.$$

Problem 31-83

From Eq. 31-4 we get $f = 1/2\pi\sqrt{LC} = 1.84$ kHz.