Physics 4B
Fall 2017

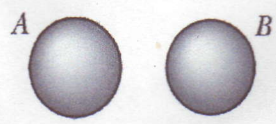
Name: Answer Key

Lab (circle one): 8:00 am   11:15 am   2:45 pm

Celebration #1: Chapters 21 – 25

Short Answer Questions (5 or 6 points each)

Question 1 (5 points)
The initial charges on two identical metal spheres are $q_A = +4.00\ \mu C$ and $q_B = -7.50\ \mu\ C$. If the two spheres are touched together, how many electrons will get transferred from sphere B to sphere A?



since spheres are identical, charge will split evenly:

$q_A = q_B = \frac{(+4.00\mu C - 7.50\mu C)}{2} = -1.75\mu C \rightarrow$ B must transfer $-5.75\ \mu C$ to A

$n = \frac{-5.75\times10^{-6}C}{-1.602\times10^{-19}C} = \boxed{3.59\times10^{13}\text{ electrons}}$

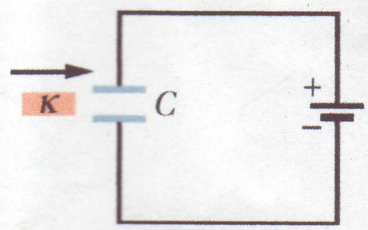
Question 2 (6 points)
A parallel plate capacitor is connected to a power supply as shown in the figure to the right. While the capacitor is still connected to the power supply, a dielectric is inserted between the plates. After the dielectric is inserted, does each of the following quantities increase, decrease or remain the same:

a) capacitance $\boxed{\text{increase}}$ $(C = KC_0)$

b) voltage across the capacitor $\boxed{\text{remain the same}}$

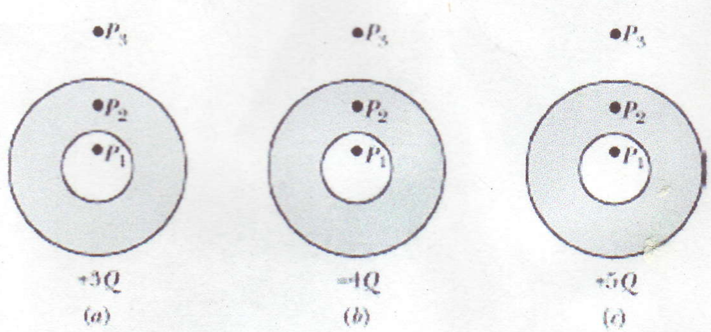
c) charge stored on the plates $\boxed{\text{increase}}$ $(q = CV)$

d) electric field between the plates $\boxed{\text{remain the same}}$



$\hookrightarrow E = V/d$ V is same so E is same

Question 3 (6 points)
Shown below are three hollow conducting spheres of the same size; the net charge of each sphere is given (+3Q, -4Q, and +5Q). Rank the spheres according to the *magnitudes* of the electric fields they produce, *from greatest to least*, at **(a)** points P_1, which are the same radial distance within the hollows; **(b)** points P_2, which are at the same radial distance within the spheres; and **(c)** points P_3, which are at the same radial distance outside the spheres. Rank the spheres according to the electric potential, *from most positive to most negative*, at **(d)** points P_1, **(e)** points P_2, and **(f)** points P_3.



a) $\boxed{\text{all tie}}$ $(E = 0)$

b) $\boxed{\text{all tie}}$ $(E = 0)$

c) $\boxed{c, b, a}$ $\left(E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)$

d) $\boxed{c, a, b}$

e) $\boxed{c, a, b}$

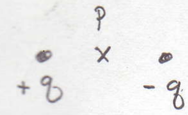
f) $\boxed{c, a, b}$

at

P_1 and $P_2 \rightarrow V = \frac{1}{4\pi\varepsilon_0} q/R$

$P_3 \rightarrow V = \frac{1}{4\pi\varepsilon_0} q/r$

Question 4 (6 points)
Are each of the following statements True or False?

for example 

at P, $V=0$ $E \neq 0$

$\boxed{F}$ **a)** If the electric potential is zero at a point, the electric field must also be zero at that point.

$\boxed{F}$ **b)** If the electric field is zero in some region of space, the electric potential must also be zero in that region. for example, inside a spherical conductor, $E=0$ but $V = \frac{1}{4\pi\varepsilon_0} q/R$

$\boxed{T}$ **c)** If the electric potential is zero in some region of space, the electric field must also be zero in that region. if $V = \text{constant}$, then $E=0$ because $E_s = -\partial V/\partial s$

$\boxed{T}$ **d)** Electric field lines always point toward regions of lower potential. $\vec{E}$ points from higher to lower potential

$\boxed{T}$ **e)** In electrostatics, the surface of a conductor is an equipotential surface.

$\boxed{T}$ **f)** Physics rules! (careful how you answer – it's worth 1 point ☺)

Question 5 (5 points)
Use the binomial expansion to show that the expression for the electric field a distance z above a charged circular disk of radius R:

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

reduces to the equation for a point charge ($E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}$) when z → ∞. Show all steps of your work.

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - z(z^2+R^2)^{-1/2}\right]$$

$$(z^2+R^2)^{-1/2} = (z^2)^{-1/2}(1+R^2/z^2)^{-1/2}$$

$$= (z^{-1})(1+R^2/z^2)^{-1/2} \approx (z^{-1})\left(1 - \frac{R^2}{2z^2}\right)$$

$$(z^2+R^2)^{-1/2} \approx z^{-1}\left(1 - \frac{R^2}{2z^2}\right)$$

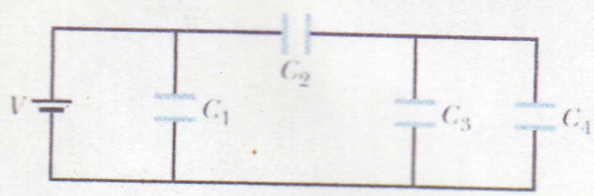
$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - z(z^{-1})\left(1-\frac{R^2}{2z^2}\right)\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - (1 - R^2/2z^2)\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\frac{R^2}{2z^2}\right) \qquad \sigma = q/A = q/\pi R^2$$

$$E = \frac{(q/\pi R^2)}{2\varepsilon_0}\left(\frac{R^2}{2z^2}\right) \longrightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}}$$

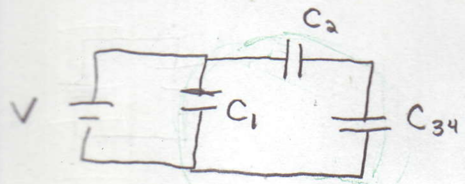
Problem 1

In the figure below, V = 12 V, $C_1 = C_2 = 2.0\ \mu F$, $C_3 = 1.0\ \mu F$, and $C_4 = 3.0\ \mu F$. What are the charge on and the voltage across each capacitor?



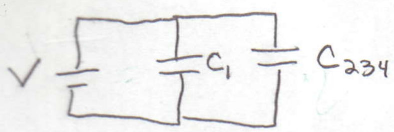
C_3 & C_4 in parallel:

$$C_{34} = C_3 + C_4 = \underline{\underline{4.0\,\mu F}}$$

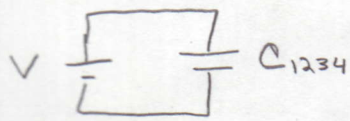


C_2 + C_{34} in series: $\dfrac{1}{C_{234}} = \dfrac{1}{C_2} + \dfrac{1}{C_{34}}$

$$\frac{1}{C_{234}} = \frac{1}{2.0\,\mu F} + \frac{1}{4.0\,\mu F} \longrightarrow C_{234} = \underline{\underline{1.33\,\mu F}}$$



C_1 + C_{234} in parallel: $C_{1234} = C_1 + C_{234} = \underline{\underline{3.33\,\mu F}}$



$$V_{1234} = 12V \longrightarrow V_1 = V_{234} = V_{1234} = 12V$$

$\boxed{V_1 = 12V}$ $\quad q_1 = C_1 V_1 = (2.0\,\mu F)(12V) \longrightarrow \boxed{q_1 = 24\,\mu C}$

$$V_{234} = 12V \longrightarrow q_{234} = C_{234} V_{234} = (1.33\,\mu F)(12V) = 16\,\mu C$$

$q_2 = q_{34} = q_{234} = 16\,\mu C \quad \boxed{q_2 = 16\,\mu C}$

$V_2 = q_2 / C_2 = 16\,\mu C / 2.0\,\mu F \longrightarrow \boxed{V_2 = 8.0V}$

$q_{34} = 16\,\mu C \quad V_{34} = q_{34}/C_{34} = 16\,\mu C / 4.0\,\mu F = 4.0V$

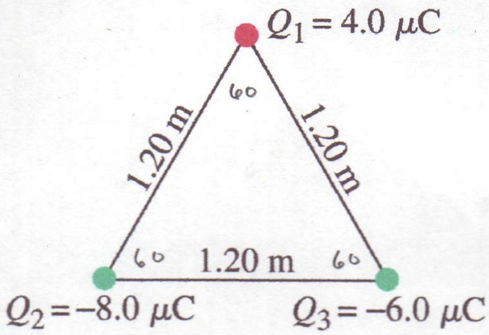
$$V_3 = V_4 = V_{234} = 4.0V$$

$\boxed{V_3 = 4.0V}$ $\quad q_3 = C_3 V_3 = (1.0\,\mu F)(4.0V) \longrightarrow \boxed{q_3 = 4.0\,\mu C}$

$\boxed{V_4 = 4.0V}$ $\quad q_4 = C_4 V_4 = (3.0\,\mu F)(4.0V) \longrightarrow \boxed{q_4 = 12\,\mu C}$

Problem 2

Three charged particles are placed at the corners of an equilateral triangle of side 1.20 m (see the figure below). The charges are $Q_1 = +4.0\ \mu C$, $Q_2 = -8.0\ \mu C$, and $Q_3 = -6.0\ \mu C$. **(a)** Calculate the magnitude and direction of the net force on charge Q_1. **(b)** Calculate the magnitude and direction of the net force on charge Q_2.

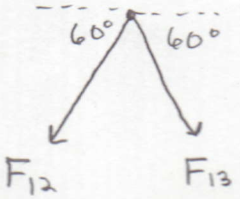


a) $$F_{12} = \frac{K|q_1||q_2|}{r^2} = \frac{(8.99\times10^9\ Nm^2/C^2)(4.0\times10^{-6}C)(8.0\times10^{-6}C)}{(1.20m)^2}$$

$$F_{12} = 0.20N$$

$$F_{13} = \frac{(8.99\times10^9\ Nm^2/C^2)(4.0\times10^{-6}C)(6.0\times10^{-6}C)}{(1.20m)^2}$$

$$F_{13} = 0.15N$$



$$\Sigma F_x = F_{13}\cos 60° - F_{12}\cos 60° = -0.025N$$

$$\Sigma F_y = -F_{12}\sin 60° - F_{13}\sin 60° = -0.303N$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} \longrightarrow \boxed{|\vec{F}| = 0.304N}$$

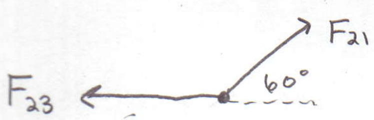
$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = 85.3° \rightarrow \text{wrong quadrant}$$

$$\boxed{\theta = 265°}$$

(b)

$$F_{21} = F_{12} = 0.20N$$

$$F_{23} = \frac{(8.99\times10^9\ Nm^2/C^2)(8.0\times10^{-6}C)(6.0\times10^{-6}C)}{(1.20m)^2} = 0.30N$$



$$\Sigma F_x = F_{21}\cos 60° - F_{23} = -0.20N$$

$$\Sigma F_y = F_{21}\sin 60° = 0.173N$$

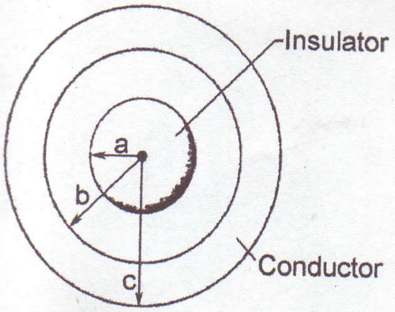
$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \boxed{0.26N}$$

$$\theta = \tan^{-1}(F_y/F_x) = -40.8° \rightarrow \text{wrong quadrant} \quad \boxed{\theta = 139°}$$

Problem 3

A solid, insulating sphere of radius a has a uniform charge density ρ and a total charge Q. Concentric with this sphere is an *uncharged, conducting shell* whose inner and outer radii are b and c (c > b > a). **Use Gauss' law to derive** the magnitudes of the electric field: **(Show all of your work!)**

a) inside the insulating sphere (r < a)
b) in-between the sphere and the shell (a < r < b)
c) inside the conducting shell (b < r < c)
d) outside the shell (r > c).
e) Make a plot of the magnitude of the electric field versus r for all of the regions.



charge on inside of conducting shell = −Q
charge on outside of conducting shell = +Q

a) $\oint \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0$

$\oint E\, dA \cos\theta = q_{encl}/\varepsilon_0 \qquad q_{encl} = \left(\frac{V_{encl}}{V}\right) Q$

$E \oint dA = \left(r^3/a^3\, Q\right)/\varepsilon_0 \qquad = r^3/a^3\, Q$

$E(4\pi r^2) = \left(\frac{r^3}{a^3}\right)\left(Q/\varepsilon_0\right) \longrightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{Qr}{a^3}}$ or $E = \frac{\rho r}{3\varepsilon_0}$ where $\rho = \frac{Q}{(4/3\pi a^3)}$

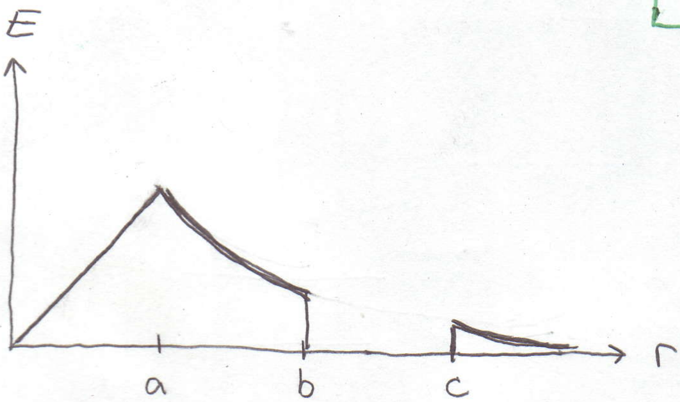
(b) $\oint \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0 \longrightarrow \oint E\, dA \cos\theta = q_{encl}/\varepsilon_0 \longrightarrow E \oint dA = q_{encl}/\varepsilon_0$

$E(4\pi r^2) = Q/\varepsilon_0 \longrightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}}$

(c) $\oint \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0 \longrightarrow E \oint dA = q_{encl}/\varepsilon_0 \quad q_{encl} = 0 \longrightarrow \boxed{E = 0}$

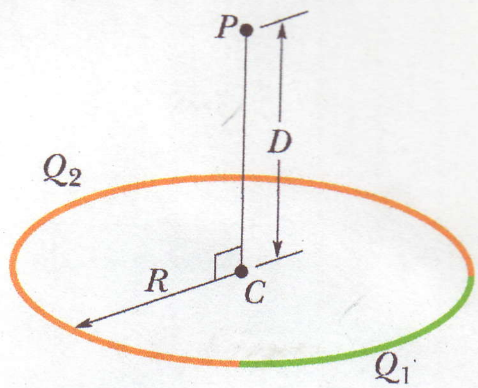
(d) same steps as part b, $q_{encl} = Q \longrightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}}$

(e)



Problem 4

A plastic rod has been bent into a circle of radius $R = 8.20$ cm. It has a charge $Q_1 = +4.20$ pC uniformly distributed along one quarter of its circumference and a charge $Q_2 = -6Q_1$ uniformly distributed along the rest of the circumference (see figure below). With $V = 0$ at infinity, what is the electric potential at **(a)** the center C of the circle and **(b)** point P, on the central axis of the circle at distance $D = 6.71$ cm from the center?



a) $$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$ $r = R$ for all points

$$V = \frac{1}{4\pi\varepsilon_0}\frac{1}{R}\int dq$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{R}(Q_1 + Q_2)$$ $Q_2 = -6Q_1$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{-5Q_1}{R} = (8.99\times10^9\,Nm^2/C^2)\frac{(-5)(4.20\times10^{-12}\,C)}{0.082\,m}$$

$$\boxed{V = -2.30\,V}$$

(b) $$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$ $r = \sqrt{R^2 + D^2}$ for all points

$$V = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{R^2+D^2}}\int dq = \frac{1}{4\pi\varepsilon_0}\frac{-5Q}{\sqrt{R^2+D^2}}$$

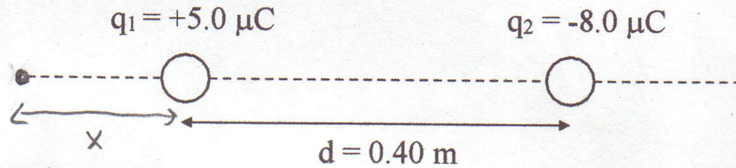
$$V = \frac{(8.99\times10^9\,Nm^2/C^2)(-5)(4.20\times10^{-12}\,C)}{\sqrt{(0.082\,m)^2 + (0.0671\,m)^2}}$$

$$\boxed{V = -1.78\,V}$$

Problem 5

Two charged conducting shells (fixed in place) of are separated by d = 0.40 m and have charges of $q_1 = +5.0\ \mu C$ and $q_2 = -8.0\ \mu C$ as shown in the figure below.

only place we can have $E = 0$ is to the left of q_1



a) Find the point in space along the line containing the two spheres (not at infinity) where the electric field is zero.

$$\sum \vec{E} = 0 \longrightarrow -\frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{x^2} + \frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{(x+d)^2} = 0$$

$$\frac{|q_1|}{x^2} = \frac{|q_2|}{(x+d)^2} \longrightarrow q_1(x+d)^2 = q_2 x^2 \longrightarrow q_1(x^2+2xd+d^2) - q_2x^2 = 0$$

$$(q_1 - q_2)x^2 + 2q_1 d x + q_1 d^2 = 0$$

$$(5.0\mu C - 8.0\mu C)x^2 + 2(5.0\mu C)(0.40m)x + (5.0\mu C)(0.40m)^2 = 0$$

$$(-3.0\mu C)x^2 + (4.0\mu C\cdot m)x + (0.80\mu C\cdot m^2) = 0$$

$$(-3.0)x^2 + (4.0m)x + (0.80m^2) = 0$$

$$x = \frac{-4.0m \pm \sqrt{(4.0m)^2 - 4(-3.0)(0.80m^2)}}{2(-3.0)} \longrightarrow \boxed{x = 1.51m \text{ to left of } q_1}$$

b) A proton ($m = 1.67 \times 10^{-27}$ kg) is released from rest 0.20 m to the left of the +5.0 μC sphere. What is the speed of the proton a very long time later?

from conservation of energy $\longrightarrow K_f + \underbrace{U_f}_{=0} = \underbrace{K_i}_{=0} + U_i$

$$K_f = U_i \longrightarrow \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_P}{0.20m} + \frac{q_2 q_P}{0.60m}\right)$$

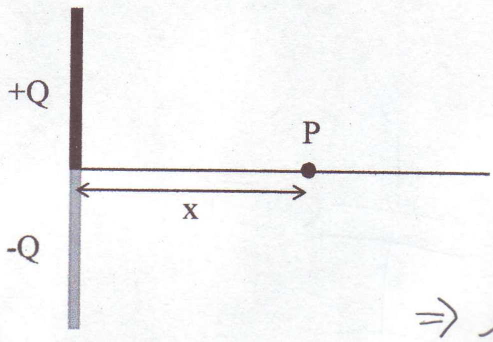
$$v = \left[\frac{2}{m}\frac{1}{4\pi\varepsilon_0}q_P\left(\frac{q_1}{0.20m} + \frac{q_2}{0.60m}\right)\right]^{1/2}$$

$$v = \left[\frac{2}{(1.67\times10^{-27}Kg)}\left(8.99\times10^9\frac{Nm^2}{C^2}\right)(1.602\times10^{-19}c)\left(\frac{5.0\times10^{-6}C}{0.20m} - \frac{8.0\times10^{-6}C}{0.60m}\right)\right]^{1/2}$$

$$\boxed{v = 4.49\times10^6 m/s}$$

Problem 6

A thin rod of length L is placed along the y-axis such that it's two ends lie at (0, -L/2) and (0, L/2) as shown in the figure below. A charge of +Q is uniformly distributed along the positive half of the rod and a charge of -Q is uniformly distributed along the negative half of the rod. Find the magnitude and direction of the electric field at the point P, a distance x from the center of the rod. Express your answer in terms of Q, L, and x.



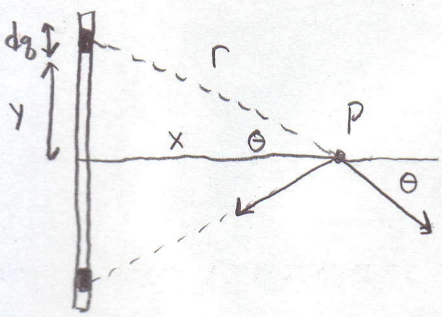
Possible useful integrals:

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = -\frac{1}{(x^2+a^2)^{1/2}}$$

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$



$\Rightarrow$ horizontal components cancel ($dE\cos\theta$), vertical components add ($dE\sin\theta$)

$$E = \int dE_y = \int dE\sin\theta = \frac{1}{4\pi\varepsilon_0}\int \frac{dq\sin\theta}{r^2}$$

$$dq = \lambda\,dy$$

$$\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}} \qquad E = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\,dy\,y}{(x^2+y^2)^{3/2}}$$

$$r^2 = x^2+y^2$$

$$E = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L/2}^{L/2} \frac{y\,dy}{(x^2+y^2)^{3/2}} = \frac{2\lambda}{4\pi\varepsilon_0}\int_0^{L/2} \frac{y\,dy}{(x^2+y^2)^{3/2}} \qquad \text{integral} = -\frac{1}{(x^2+y^2)^{1/2}}$$

$$E = -\frac{2\lambda}{4\pi\varepsilon_0}\frac{1}{\sqrt{x^2+y^2}}\Bigg|_0^{L/2} = -\frac{\lambda}{2\pi\varepsilon_0}\left[\frac{1}{\sqrt{x^2+L^2/4}} - \frac{1}{\sqrt{x^2}}\right]$$

$$E = \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{1}{x} - \frac{1}{\sqrt{x^2+L^2/4}}\right] \qquad \lambda = Q/L = Q/(L/2) = 2Q/L$$

$$\boxed{\vec{E} = \frac{Q}{L\pi\varepsilon_0}\left[\frac{1}{x} - \frac{1}{\sqrt{x^2+L^2/4}}\right](-\hat{j})}$$