Physics 4B
Fall 2017

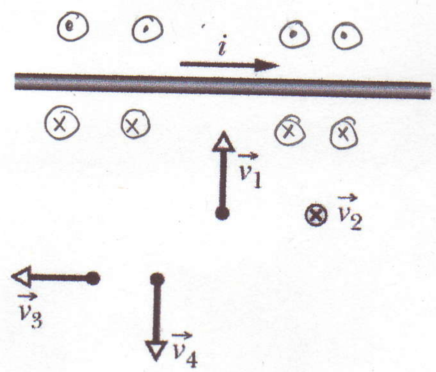
Name: Answer Key

Lab (circle one): 8:00 am 11:15 am 2:45 pm

Celebration #2: Circuits and Magnetism

Short Answer Questions (28 points total)

Question 1 (6 points)
The figure to the right shows a snapshot of the velocity vectors of four *electrons* near a wire carrying current i. The four velocity vectors have the same magnitude; velocity $\vec{v}_2$ is directed into the page. Particle 1 and 2 are at the same distance from the wire, as are particles 3 and 4.



a) What is the direction of the magnetic force (if any) on each electron?

Particle 1: right

Particle 3: up

Particle 2: no force

Particle 4: left

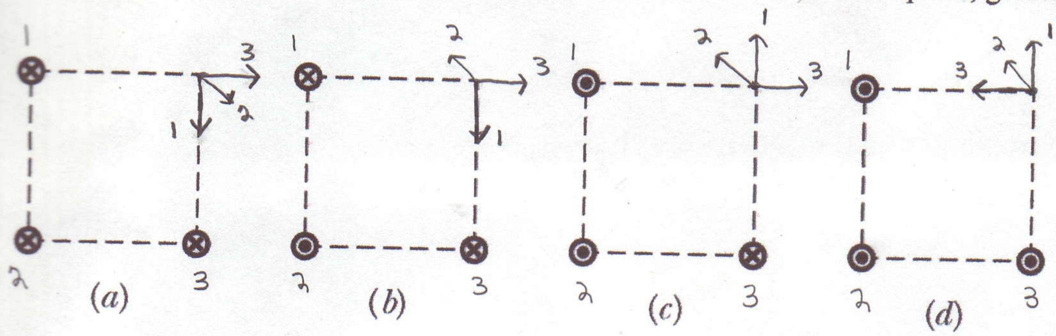
b) Rank the electrons according to the magnitudes of the magnetic forces on them due to current i, greatest first.

$F = q\vec{v}\times\vec{B} = qvB\sin\theta$

$B = \frac{\mu_0 i}{2\pi R}$

1, 3 + 4 tie, 2 (zero)

Question 2 (5 points)
As shown below, three long wires, with identical current either directly into or directly out of the page, form three partial squares. Rank the squares according to the magnitude of the net magnetic field produced by the wires at the (empty) upper right corner of the square, greatest first.



a + d tie, c, b

Question 3 (6 points)
A potential difference V is connected across a device with resistance R, causing current *i* through the device. Rank the following variations according to the change in the rate at which electrical energy is converted to thermal energy, greatest change first: (a) V is doubled with R unchanged, (b) *i* is doubled with R unchanged, (c) R is doubled with V unchanged, (d) R is doubled with *i* unchanged.

$P = iV$
$= i^2 R$
$= V^2/R$

(a) $P = (2V)^2/R = 4V^2/R$

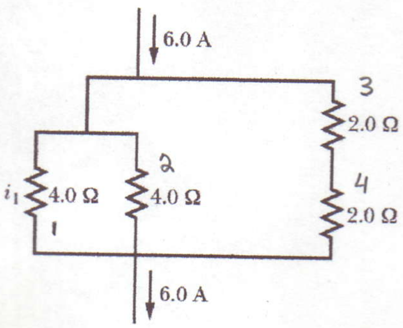
(b) $P = (2i)^2 R = 4i^2 R$

(c) $P = V^2/(2R) = \frac{1}{2} V^2/R$

(d) $P = i^2(2R) = 2i^2 R$

$\boxed{a + b \text{ tie, } d, c}$

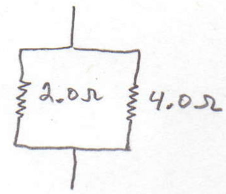
Question 4 (5 points)
The figure below shows a portion of a circuit. What is the magnitude of current i_1?



3 + 4 in series $\rightarrow R_{34} = 4.0\,\Omega$

1 + 2 in parallel $\rightarrow R_{12} = 2.0\,\Omega$

12 + 34 in parallel $\rightarrow R_{1234} = 1.33\,\Omega$



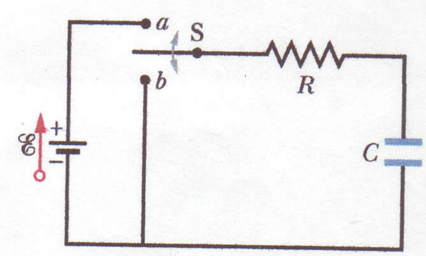
$V_{1234} = i_{1234} R_{1234} = (6.0\text{A})(1.33\,\Omega) = 8.0\text{V}$

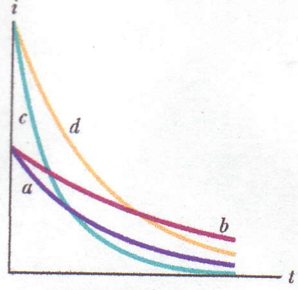
$V_{12} = V_{34} = V_{1234} = 8.0\text{V}$

$V_1 = V_2 = V_{12} = 8.0\text{V}$

$i_1 = V_1/R_1 = \frac{8.0\text{V}}{4.0\,\Omega} \rightarrow \boxed{i_1 = 2.0\text{A}}$

Question 5 (6 points)
After the switch in the figure to the right is closed on point a, there is a current i through resistance R. The figure below gives the current for four sets of values of R and capacitance C: (1) R_0 and C_0, (2) $2R_0$ and C_0, (3) R_0 and $2C_0$, (4) $2R_0$ and $2C_0$. Which set goes with which curve?





(1) $\boxed{c}$

(2) $\boxed{a}$

(3) $\boxed{d}$

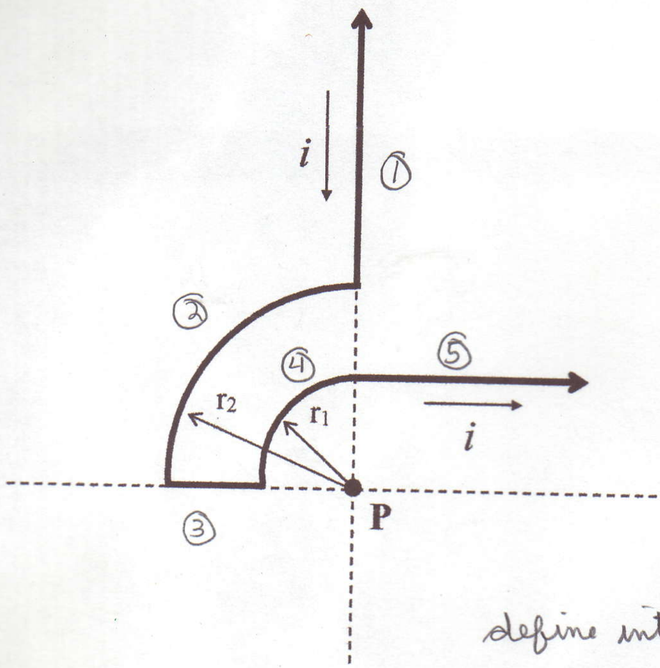
(4) $\boxed{b}$

Problems (12 points each)

Problem 1

For the current carrying wire shown in the figure below, i = 25.1 mA, r_1 = 5.0 cm, and r_2 = 10.00 cm. Find the magnitude and direction of the magnetic field at point P.

(**Note**: the top vertical wire and the right horizontal wire are semi-infinite wires.)



$$B_1 = B_3 = 0$$

$$B_2 = \frac{\mu_0 i \phi}{4\pi R} \text{ out of page} \quad \phi = \pi/2, \; R = r_2$$

$$B_4 = \frac{\mu_0 i \phi}{4\pi R} \text{ into page} \quad \phi = \pi/2, \; R = r_1$$

$$B_5 = \frac{\mu_0 i}{4\pi r} \text{ into page} \quad r = r_1$$

define into the page as + :

$$B_{net} = B_4 + B_5 - B_2$$

$$B_{net} = \frac{\mu_0 i (\pi/2)}{4\pi r_1} + \frac{\mu_0 i}{4\pi r_1} - \frac{\mu_0 i (\pi/2)}{4\pi r_2} = \frac{\mu_0 i}{4\pi}\left(\frac{\pi}{2r_1} + \frac{1}{r_1} - \pi/2r_2\right)$$

$$B_{net} = \frac{(1.26\times10^{-6}\, Tm/A)(25.1\times10^{-3} A)}{4\pi}\left[\frac{\pi}{2(0.050m)} + \frac{1}{0.050m} - \frac{\pi}{2(0.10m)}\right]$$

$$\boxed{\vec{B}_{net} = 9.0\times10^{-8}\ T \text{ into the page}}$$

$B_2 = 3.95\times10^{-8}$ T ⊙

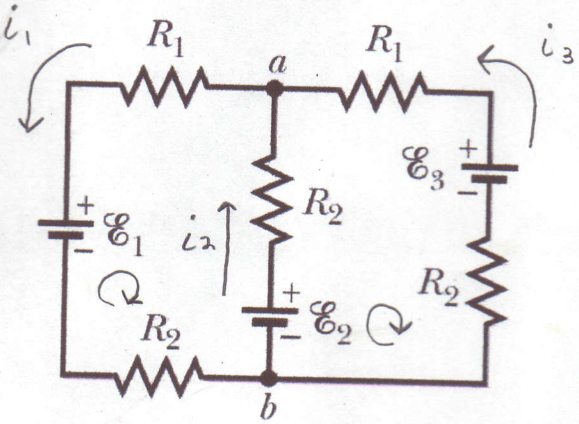
$B_4 = 7.91\times10^{-8}$ T ⊗

$B_5 = 5.03\times10^{-8}$ T ⊗

Problem 2

In the figure below, assume that $R_1 = 5.0\ \Omega$, $R_2 = 10.0\ \Omega$, $\varepsilon_1 = 2.0$ V, $\varepsilon_2 = \varepsilon_3 = 4.0$ V. **(a)** Calculate the current through each ideal battery. **(b)** What is $V_{ab} = V_a - V_b$?

(**Note**: clearly indicate the direction of all currents and loops.)



from junction rule at a:

$$i_1 = i_2 + i_3 \quad (1)$$

from left loop:

$$\mathcal{E}_1 + i_1 R_1 + i_2 R_2 - \mathcal{E}_2 + i_1 R_2 = 0$$

$$2.0V + i_1(5.0\Omega) + i_2(10.0\Omega) - 4.0V + i_1(10.0\Omega) = 0$$

$$15 i_1 + 10 i_2 - 2 = 0 \quad (2)$$

from right loop:

$$\mathcal{E}_2 - i_2 R_2 + i_3 R_1 - \mathcal{E}_3 + i_3 R_2 = 0$$

$$4.0V - i_2(10.0\Omega) + i_3(5.0\Omega) - 4.0V + i_3(10.0\Omega) = 0 \longrightarrow 15 i_3 - 10 i_2 = 0 \quad (3)$$

$$\left.\begin{array}{ll} \text{from (3)} & 15 i_3 = 10 i_2 \longrightarrow i_3 = 10/15\, i_2 \\ \text{from (2)} & 15 i_1 = 2 - 10 i_2 \longrightarrow i_1 = 2/15 - 10/15\, i_2 \end{array}\right\} \text{put into (1)}$$

$$i_1 = i_2 + i_3 \longrightarrow (2/15 - 10/15\, i_2) = i_2 + (10/15)\, i_2$$

$$2/15 = i_2 + \frac{20}{15} i_2 \longrightarrow 2/15 = \frac{35}{15} i_2 \longrightarrow \boxed{\begin{array}{l} i_2 = 0.057A \\ \quad = 57mA \end{array}}$$

$$i_3 = 10/15\, i_2 = (10/15)(0.057A) \longrightarrow \boxed{i_3 = 0.038A = 38\ mA}$$

$$i_1 = i_2 + i_3 \longrightarrow \boxed{i_1 = 0.095A = 95\ mA}$$

(b) $V_a + i_2 R_2 - \mathcal{E}_2 = V_b \longrightarrow V_a - V_b = \mathcal{E}_2 - i_2 R_2$

$$V_a - V_b = 4.0V - (0.057A)(10.0\Omega)$$

$$\boxed{V_a - V_b = 3.43\ V}$$

Problem 3

The current-density in a certain wire of radius r = 3.00 mm is given by $J = (2.75\times10^{10}\ A/m^4)r^2$, where r is the radial distance from the center of the wire. The potential difference across the ends of the wire is 60.0 V. How much energy is converted to thermal energy in the wire in 1.50 hours?

$$i = \int \vec{J}\cdot d\vec{A} \qquad J = (2.75\times10^{10}\ A/m^4)\,r^2 = J_o r^2$$

$$dA = 2\pi r\,dr$$

$$i = \int_0^r (J_o r^2)(2\pi r\,dr)\cos 0^\circ = 2\pi J_o \int_0^r r^3\,dr = 2\pi J_o \left(\frac{r^4}{4}\right)\Big|_0^r$$

$$i = \pi/2\ J_o r^4$$

$$i = (\pi/2)(2.75\times10^{10}\ A/m^4)(0.0030\,m)^4 \longrightarrow \underline{i = 3.50\,A}$$

$$P = iV = (3.50\,A)(60.0\,V) = \underline{210\,W}$$

$$P = E/t \longrightarrow E = Pt \qquad t = 1.50\ \text{hours} = \underline{5400\,s}$$

$$E = (210\,W)(5400\,s) \longrightarrow \boxed{E = 1.13\times10^6\,J}$$

Problem 4

a) An electron moving with a velocity $\vec{v} = (5.0\times10^4\,\text{m/s})\hat{i} - (6.5\times10^4\,\text{m/s})\hat{k}$ enters a region of space that contains both a magnetic field and an electric field. The magnetic field is given by $\vec{B} = (10.6\,\text{mT})\hat{j} + (9.5\,\text{mT})\hat{k}$ and the electric field is given by $\vec{E} = (935N/C)\hat{i} - (322N/C)\hat{j} - (590N/C)\hat{k}$. Determine the net force on the electron.

$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

$$\vec{v}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5.0\times10^4\,\text{m/s} & 0 & -6.5\times10^4\,\text{m/s} \\ 0 & 10.6\times10^{-3}\,\text{T} & 9.5\times10^{-3}\,\text{T} \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} 0 & -6.5\times10^4\,\text{m/s} \\ 10.6\times10^{-3}\,\text{T} & 9.5\times10^{-3}\,\text{T} \end{vmatrix} - \hat{j}\begin{vmatrix} 5.0\times10^4\,\text{m/s} & -6.5\times10^4\,\text{m/s} \\ 0 & 9.5\times10^{-3}\,\text{T} \end{vmatrix} + \hat{k}\begin{vmatrix} 5.0\times10^4\,\text{m/s} & 0 \\ 0 & 10.6\times10^{-3}\,\text{T} \end{vmatrix}$$

$$= (689\,\text{Tm/s})\hat{i} - (475\,\text{Tm/s})\hat{j} + (530\,\text{Tm/s})\hat{j}$$

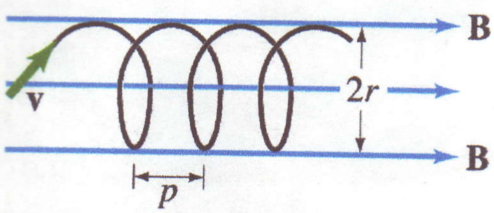
$q_e = -1.602\times10^{-19}\,\text{C}$

$$\vec{F}_B = q(\vec{v}\times\vec{B}) = (-1.10\times10^{-16}\,\text{N})\hat{i} + (7.61\times10^{-17}\,\text{N})\hat{j} - (8.49\times10^{-17}\,\text{N})\hat{k}$$

$$\vec{F}_E = q\vec{E} = (-1.50\times10^{-16}\,\text{N})\hat{i} + (5.16\times10^{-17}\,\text{N})\hat{j} + (9.45\times10^{-17}\,\text{N})\hat{k}$$

$$\vec{F}_{net} = \vec{F}_E + \vec{F}_B = \boxed{(-2.60\times10^{-16}\,\text{N})\hat{i} + (1.28\times10^{-16}\,\text{N})\hat{j} + (9.60\times10^{-18}\,\text{N})\hat{k}}$$

b) An electron ($m = 9.11\times10^{-31}$ kg) enters a uniform magnetic field B = 0.750 T at a 45.0° angle to **B** as shown in the figure below. Determine the radius r and pitch p of the electron's helical path assuming its speed is 3.50×10^6 m/s.



$r = \dfrac{mv_\perp}{qB}$ $\quad v_\perp = v\sin\theta$

$p = v_{\parallel}T$ $\quad T = \dfrac{2\pi m}{qB}$ $\quad v_{\parallel} = v\cos\theta$

$m = 9.11\times10^{-31}\,\text{Kg}$

$|q| = 1.602\times10^{-19}\,\text{C}$

$\vec{B} = 0.750\,\text{T}$

$\theta = 45.0°$

$v = 3.50\times10^6\,\text{m/s}$

$$r = \frac{mv\sin\theta}{qB} = \frac{(9.11\times10^{-31}\,\text{Kg})(3.50\times10^6\,\text{m/s})\sin 45°}{(1.602\times10^{-19}\,\text{C})(0.750\,\text{T})}$$

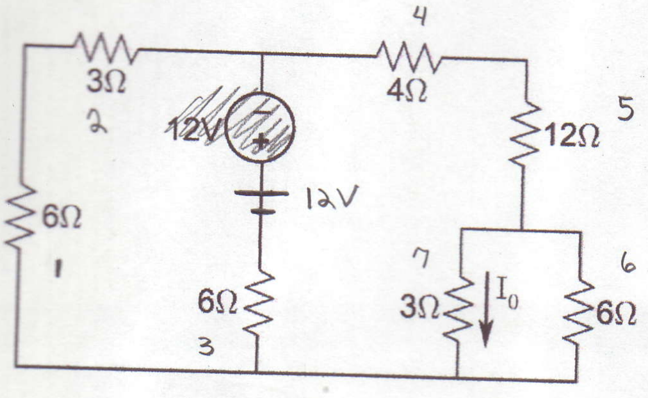
$$\boxed{r = 1.88\times10^{-5}\,\text{m}}$$

$$p = (v\cos\theta)\left(\frac{2\pi m}{qB}\right) = \frac{(3.50\times10^6\,\text{m/s})\cos 45.0°(2\pi)(9.11\times10^{-31}\,\text{Kg})}{(1.602\times10^{-19}\,\text{C})(0.750\,\text{T})}$$

$$\boxed{p = 1.18\times10^{-4}\,\text{m}}$$

Problem 5

(a) In the circuit shown determine the equivalent resistance R_{eq}. (b) Determine the current I_0 through the indicated resistor.



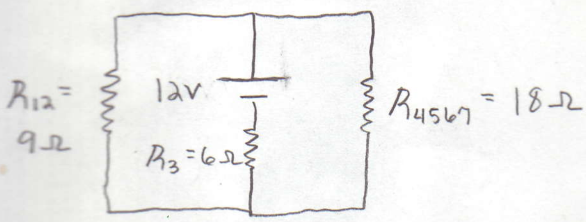
$R_1 + R_2$ in series:

$R_{12} = 6\Omega + 3\Omega \longrightarrow \underline{R_{12} = 9\Omega}$

$R_6 + R_7$ in parallel:

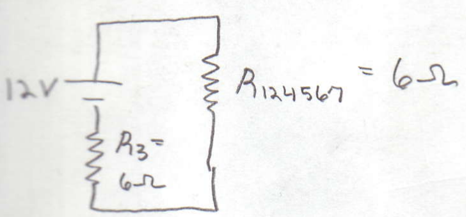
$\frac{1}{R_{67}} = \frac{1}{6\Omega} + \frac{1}{3\Omega} \longrightarrow \underline{R_{67} = 2\Omega}$

R_4, R_5, and R_{67} in series: $R_{4567} = 4\Omega + 12\Omega + 2\Omega \longrightarrow \underline{R_{4567} = 18\Omega}$



R_{12} and R_{4567} in parallel:

$\frac{1}{R_{124567}} = \frac{1}{9\Omega} + \frac{1}{18\Omega} \longrightarrow \underline{R_{124567} = 6\Omega}$



R_3 and R_{124567} in series:

$R_{3124567} = R_3 + R_{124567} = 6\Omega + 6\Omega = \underline{12\Omega}$

$\boxed{R_{eq} = 12\Omega}$

$i_{3124567} = \frac{V_{3124567}}{R_{3124567}} = \frac{12V}{12\Omega} = 1.0A \longrightarrow \underline{i_{124567} = 1.0A}$

$V_{124567} = i_{124567} R_{124567} = (1.0A)(6\Omega) = 6.0V \longrightarrow \underline{V_{4567} = 6.0V}$

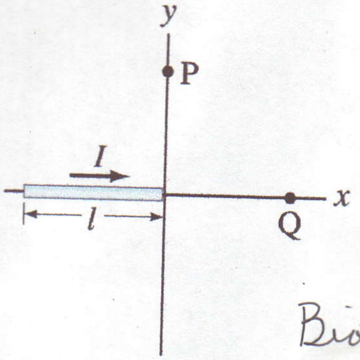
$i_{4567} = \frac{V_{4567}}{R_{4567}} = \frac{6.0V}{18\Omega} = 0.33A \longrightarrow \underline{i_{67} = 0.33A}$

$V_{67} = i_{67} R_{67} = (0.33A)(2\Omega) = 0.67V \longrightarrow \underline{V_7 = 0.67V}$

$i_7 = V_7 / R_7 = \frac{0.67V}{3\Omega} \longrightarrow \boxed{i = 0.22A}$

Problem 6

A segment of wire of length l = 35.0 cm carries a current I = 1.25 A as shown in the figure below. Point Q is 42.0 cm to the right of the wire and point P is 42.0 cm above the wire. **(a)** Show that the magnetic field at point Q is zero. **(b)** What is the magnitude and direction of the magnetic field at point P. (**Note**: you must start with the Biot-Savart law and show all work.)



Possible useful integrals:

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right)$$

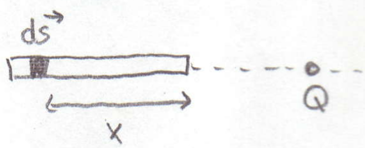
$$\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = -\frac{1}{(x^2+a^2)^{1/2}}$$

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$$

Biot-Savart

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$ direction given by $d\vec{s}\times\vec{r}$

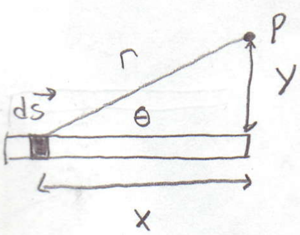
(a)



$$B = \int dB = \frac{\mu_0 i}{4\pi}\int\frac{ds\sin\theta}{r^2}$$

for point Q, $\theta = 0°$ for all segments of wire so $B = \frac{\mu_0 i}{4\pi}\int\frac{ds\sin 0°}{r^2} = 0$

(b)



$$B = \frac{\mu_0 i}{4\pi}\int\frac{ds\sin\theta}{r^2}$$ direction is out of page ⊙

$ds \to dx$ $\quad r^2 = x^2+y^2$ $\quad \sin\theta = \frac{y}{\sqrt{x^2+y^2}}$

$$B = \frac{\mu_0 i}{4\pi}\int\frac{dx\,y}{(x^2+y^2)^{3/2}} = \frac{\mu_0 i y}{4\pi}\int_0^L\frac{dx}{(x^2+y^2)^{3/2}}$$

$$B = \frac{\mu_0 i y}{4\pi}\left.\frac{x}{y^2(x^2+y^2)^{1/2}}\right|_0^L = \frac{\mu_0 i}{4\pi y}\frac{L}{(L^2+y^2)}$$

$L = 0.35\text{ m}$
$y = 0.42\text{ m}$
$i = 1.25\text{A}$

$$B = \frac{(1.26\times10^{-6}\text{ Tm/A})(1.25\text{A})}{4\pi(0.42\text{m})}\frac{(0.35\text{m})}{\sqrt{(0.35\text{m})^2+(0.42\text{m})^2}} \rightarrow \boxed{\vec{B} = 1.91\times10^{-7}\text{ T out of page}}$$