Review Problems for the Final Exam

Problem 1

$\mathcal{E} = \mathcal{E}_m \sin \omega_d t$

$i = I \sin(\omega_d t - \phi)$ $\quad \boxed{I = \mathcal{E}_m / Z = \mathcal{E}_m / \sqrt{R^2 + (X_L - X_C)^2}}$ $\quad \tan\phi = \frac{X_L - X_C}{R}$

from i we can get $V_R = iR$, $V_L = L\,di/dt$, $V_C = q/C$ $\quad q = \int i\,dt$

$\Rightarrow$ need I and ϕ

$X_L = 2\pi f_d L = 75.4\,\Omega$ $\quad X_C = \frac{1}{2\pi f_d C} = 106.1\,\Omega$ $\quad Z = 104.6\,\Omega$

$I = \mathcal{E}_m / Z = \frac{30.0\,V}{104.6\,\Omega} = 0.287\,\Omega$ $\quad \phi = \tan^{-1}\left(\frac{75.4\,\Omega - 106.1\,\Omega}{100\,\Omega}\right)$

$= -17.1°$

$\Rightarrow$ voltage across generator is a max $(30.0\,V)$ when $\sin \omega_d t = 1$

or $\omega_d t = \pi/2 = 90°$

short cut: don't solve for t $\rightarrow$ $\omega_d t - \phi = 90° - (-17.1°)$

$= 107.1°$

resistor $\rightarrow$ $V_R = iR = IR \sin(\omega_d t - \phi)$

$= (0.287\,\Omega)(100\,\Omega)\sin(107.1°)$

$$\boxed{V_R = 27.4\,V}$$

inductor $\rightarrow$ $V_L = L\, di/dt = L\, d/dt\, [I \sin(\omega_d t - \phi)]$

$= L I \omega_d \cos(\omega_d t - \phi)$

$V_L = (200\times10^{-3}\,H)(0.287A)\,2\pi(60Hz)\cos(107.1^\circ)$

$\boxed{V_L = -6.4V}$

capacitor $\rightarrow$ $V_c = q/C$ $\quad q = \int i\,dt = \int I \sin(\omega_d t - \phi)$

$= -I/\omega_d \cos(\omega_d t - \phi)$

$V_c = \dfrac{-I}{\omega_d C} \cos(\omega_d t - \phi)$

$V_c = \dfrac{-(0.287A)}{2\pi(60Hz)(25.0\times10^{-6}F)} \cos(107.1^\circ)$

$\boxed{V_c = 9.0V}$

note: $V_R + V_L + V_c = \mathcal{E}$

Problem 2

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right)$$

$$E_z = -\partial V/\partial z = \frac{-\sigma}{2\varepsilon_0}\frac{\partial}{\partial z}\left[(z^2+R^2)^{1/2} - z\right]$$

$$E_z = \frac{-\sigma}{2\varepsilon_0}\left[\frac{1}{2}(z^2+R^2)^{-1/2}(2z) - 1\right]$$

$$E_z = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

for $R >> z \longrightarrow \frac{z}{\sqrt{z^2+R^2}} \rightarrow 0 \qquad E_z = \frac{\sigma}{2\varepsilon_0}[1-0] = \sigma/2\varepsilon_0$

$\boxed{E = \sigma/2\varepsilon_0} \longrightarrow$ E from an infinite sheet

for $z >> R \qquad \frac{z}{\sqrt{z^2+R^2}} = z(z^2+R^2)^{-1/2} = z(z^2)^{-1/2}(1+R^2/z^2)^{-1/2}$

$$= z(z^{-1})(1+R^2/z^2)^{-1/2} = (1+R^2/z^2)^{-1/2}$$

for $x << 1 \rightarrow (1+x)^n \approx 1+nx$

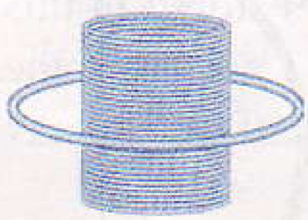
$$(1+R^2/z^2)^{-1/2} \approx 1 - R^2/2z^2$$

$E_z = \sigma/2\varepsilon_0\left[1-(1-R^2/2z^2)\right] = \frac{\sigma}{2\varepsilon_0}\frac{R^2}{2z^2} \qquad \sigma = q/A = q/\pi R^2$

$\boxed{E_z = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}} \longrightarrow$ E from a point charge

→ Final Exam review problem 3

Problem 6
In the figure below, a circular loop of wire is concentric with a solenoid and lies in a plane that is perpendicular to the solenoid's central axis. The circular loop had a radius of 6.00 cm and a resistance of 1.0 mΩ. The solenoid has a radius 2.00 cm, consists of 8000 turns per meter, and carries a current that varies with time according to i = (0.50 A/s) t. What is the induced current in the circular loop?



$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\,dA \cos 0^\circ$$

$$= B\int dA = BA$$

for a solenoid → $B = \mu_0 n i$

$A = \pi r^2$ * note r is radius of solenoid not circular loop because A is area within the $\vec{B}$-field

$$\Phi_B = \mu_0 n i (\pi r^2)$$

$\frac{d}{dt}[(0.50\,A/s)t] = (0.50\,A/s)$

$$|\mathcal{E}| = d\Phi_B/dt = \frac{d}{dt}[\mu_0 n i \pi r^2] = \mu_0 n \pi r^2\, di/dt$$

$$= \mu_0 n \pi r^2 (0.50\,A/s)$$

$$= (1.26\times10^{-6}\,Tm/A)(8000\,m^{-1})\,\pi(0.02\,m)^2(0.50\,A/s)$$

$$|\mathcal{E}| = 6.3\times10^{-6}\,V$$

$(Tm/A)(\frac{1}{m})(m^2)(A/s)$

$Tm^2/s \quad T = N/Am$

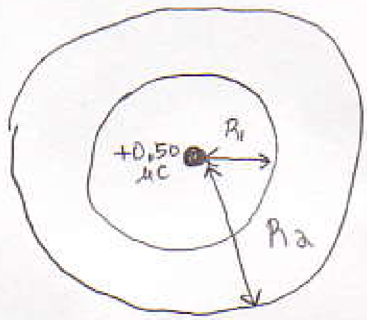
$(N/Am)(m^2/s) = \frac{Nm}{As} = \frac{J}{C} = V$

$$i = \mathcal{E}/R = \frac{6.3\times10^{-6}\,V}{1.0\times10^{-3}\,\Omega} \longrightarrow \boxed{i = 6.3\,mA}$$

Problem 4

$q_{point\ charge} = +0.50\mu C$

$q_{sphere} = +1.0\mu C$



$q_{inner} = -0.50\mu C$

$q_{outer} = 1.50\mu C$

$0 < r < R_1)$ $\oint \vec{E} \cdot d\vec{A} = q_{enc}/\varepsilon_0 \longrightarrow E(4\pi r^2) = q/\varepsilon_0$ $\quad q = q_{point\ charge}$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad q = +0.50\mu C$$

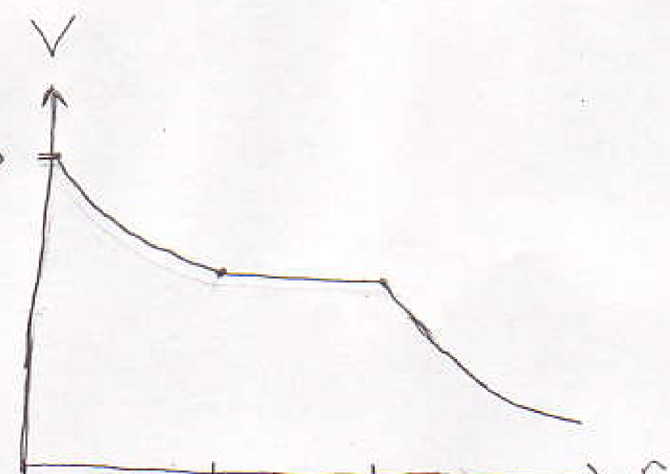
$R_1 < r < R_2)$ $E = 0$ $\quad \oint E \cdot dA = q_{enc}/\varepsilon_0$ $\quad q_{enc} = 0 = \underbrace{q_{point\ charge}}_{+0.50\mu C} + \underbrace{q_{inner}}_{-0.50\mu C}$

$r > R_2)$ $E = \frac{1}{4\pi\varepsilon_0}\frac{q_{enc}}{r^2}$ $\quad q_{enc} = q_{point\ charge} + q_{sphere} = +1.50\mu C$

$$V_{point\ charge} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = V_{sphere} \qquad E_r = \frac{-\partial V}{\partial r}$$

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \longrightarrow$

$q = +1.50\mu C$



Problem 5

$$\left.\begin{array}{l} C_2 = 1.0\mu F \\ q_2 = 10\mu F \end{array}\right\} \quad V_2 = q_2/C_2 \rightarrow \frac{10\mu C}{1.0\mu F} = 10.0V$$

$V_3 = V_2 = 10.0V$ since capacitors are in parallel

$$q_3 = C_3 V_3 = (4.0\mu F)(10.0V) = \boxed{40\mu C = q_3}$$

$$q_{23} = q_2 + q_3 = 50.0\mu C$$

$\Rightarrow$ in series, q is same $\quad q_1 = q_{23} \rightarrow \boxed{q_1 = 50\mu C}$

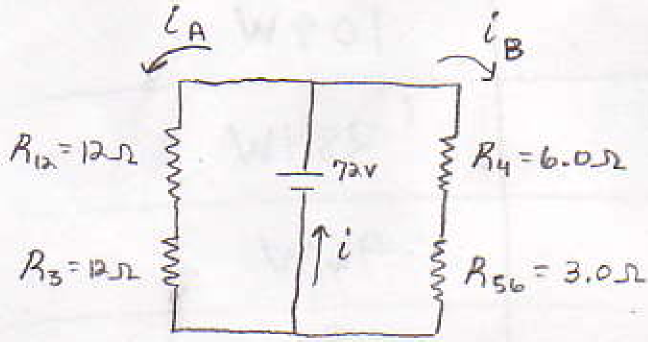
$$V_1 = q_1/C_1 = 50\mu C/2\mu F = 25V$$

$$\mathcal{E} = V_1 + V_{23} = \boxed{35V}$$

Problem 6

$$1/R_{12} = 1/R_1 + 1/R_2 \rightarrow R_{12} = 12\,\Omega$$

$$1/R_{56} = 1/R_5 + 1/R_6 \rightarrow R_{56} = 3.0\,\Omega$$



$$i_B = \frac{72V}{9.0\,\Omega} = 8.0A$$

$$i_A = \frac{72V}{24\,\Omega} = 3.0A$$

$$i = i_A + i_B = 11.0A$$

$$i_{12} = i_3 = 3.0A$$

$$V_3 = i_3 R_3 = 36V$$

$$V_{12} = i_{12} R_{12} = 36V \rightarrow V_1 = V_2 = 36V$$

$$i_1 = V_1/R_1 = 1.5A$$

$$i_2 = V_2/R_2 = 1.5A$$

$$i_4 = i_{56} = 8.0A \qquad V_{56} = i_{56} R_{56} = 24V$$

$$V_4 = i_4 R_4 = 48V$$

$$V_5 = V_6 = 24V$$

$$i_5 = V_5/R_5 = 24V/6\Omega = 4.0A$$

$$i_6 = V_6/R_6 = 24V/6\Omega = 4.0A$$

	current	V_{drop}	Power
R_1	1.5A	36V	54W
R_2	1.5A	36V	54W
R_3	3.0A	36V	108W
R_4	8.0A	48V	384W
R_5	4.0A	24V	96W
R_6	4.0A	24V	96W
$\mathcal{E}$	11.0A	72V	792W